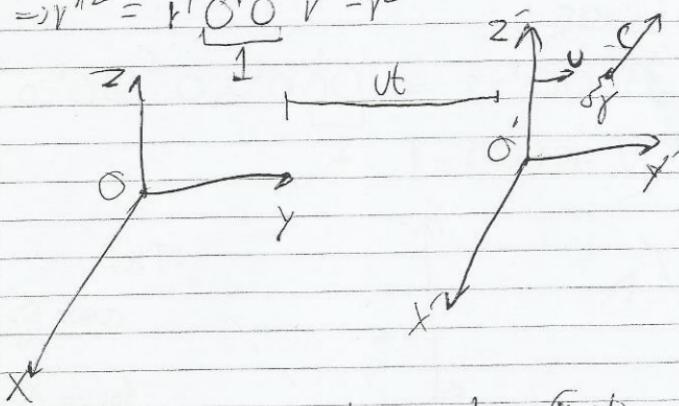


$$r^2 = x^2 + y^2 \Rightarrow r^T r = (x \ y) \begin{pmatrix} x \\ y \end{pmatrix} = x^2 + y^2$$

$$\Rightarrow r^2 = r^T \underbrace{O^T O}_I r = r^2$$



$$O: (v_x, v_y, v_z)$$

$$v_{\tilde{x}}' = \frac{dx'}{dt} = \frac{(x - vt)}{dt}$$

$$O': (v_{\tilde{x}}', v_{\tilde{y}}', v_{\tilde{z}}')$$

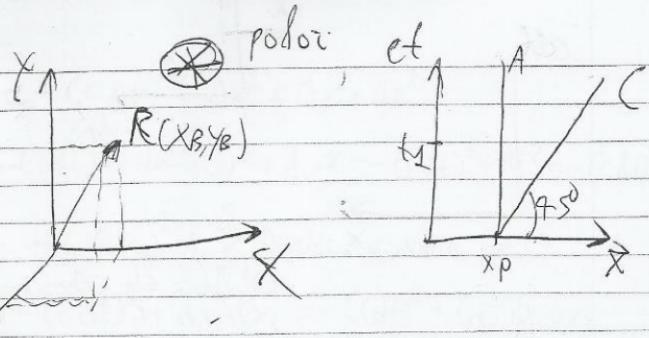
$$= \frac{dx}{dt} - v \frac{dt}{dt} = v_x - v$$

H ταχύτηρα τα γράμε αλλά c δεν είναι στα ΑΣΑ.



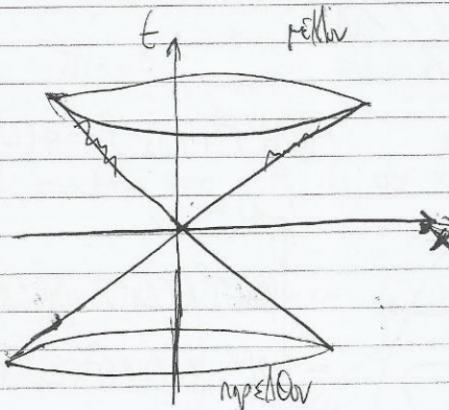
- 1) Εάν $v = 0$ πού είναι ο A και Γ σπέντε μια εκατοντάδα χιλιόμετρα
- 2) Εάν $v \neq 0$ πού είναι ο A σπέντε μια εκατοντάδα χιλιόμετρα

Μπορείτε;

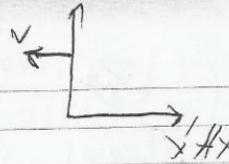


$$kd\text{tang} = \frac{C}{Vx} = \frac{Cd\ell}{dx} = \frac{d(C\ell)}{dx} \quad A: \text{vdb}\text{g}\text{r}\text{a}\text{v}\text{e}\text{m} \quad C: \text{vdb}\text{g}\text{r}\text{a}\text{v}\text{e}\text{m} = 1$$

$\Rightarrow Vx = C$

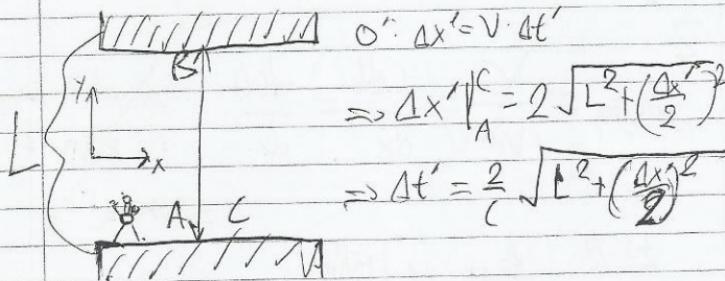


$c t$



"X-Ways Minkowski"

$$0: \Delta t = \frac{2L}{c}, \Delta x = \Delta y = \Delta z = 0$$



$$c(c\Delta t')^2 + (\Delta x')^2 = -g(L^2 + \left(\frac{\Delta x'}{2}\right)^2) + \Delta x'^2$$

$$= -4L^2$$

$$\Rightarrow -(\Delta t')^2 + (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 = -gL^2 - (c\Delta t')^2 + (\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2$$

$$-(c\Delta t')^2 + (\Delta x')^2 = ds^2$$

$$\Delta x' = v \Delta t'$$

$$\Delta y' = 0, \Delta z' = 0$$

$$\begin{pmatrix} - & + & . \\ + & + & \\ . & & + \end{pmatrix} = m_{\text{inv}}$$

$$\Delta s^2 = -(c \Delta t)^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$$

$$\begin{aligned} ds^2 &= -(cdt)^2 + dx^2 + dy^2 + dz^2 = -(cdt)^2 + dx^2 = -(cdt)^2 + \sum_{i=1}^3 dx_i^2 \\ &= -(cdt)^2 + \sum_{i=1}^3 dx_i dx_i \end{aligned}$$

$$= -(cdt)^2 + dx_i dx_i = - (cdt)^2 + \delta_{ij} dx_i dx_j$$

$$= -(cdt)^2 + \delta_{11} dx_1 dx_1 + \delta_{22} dx_2 dx_2 + \delta_{33} dx_3 dx_3$$

$$+ \delta_{21} \cancel{dx_2 dx_1} + \delta_{22} \cancel{dx_2 dx_2} + \delta_{23} \cancel{dx_2 dx_3}$$

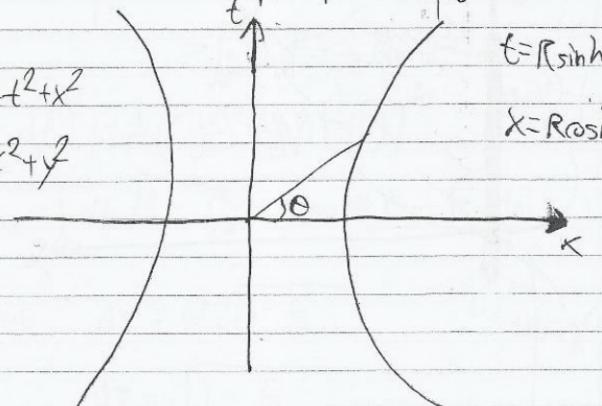
$$+ \delta_{31} \cancel{dx_3 dx_1} + \delta_{32} \cancel{dx_3 dx_2} + \delta_{33} dx_3 dx_3$$

analog $\delta_{ij} = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases} \Rightarrow \delta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$c=1 \Rightarrow m = sec \quad M_m dx_n dx_n = \gamma_{00} dx_3 dx_3 + dx_1 dx_1$$

$$R^2 = t^2 + x^2$$

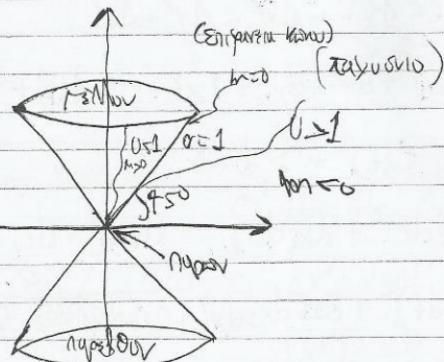
$$R^2 = x^2 + y^2$$



$$t = R \sinh \theta$$

$$x = R \cosh \theta$$

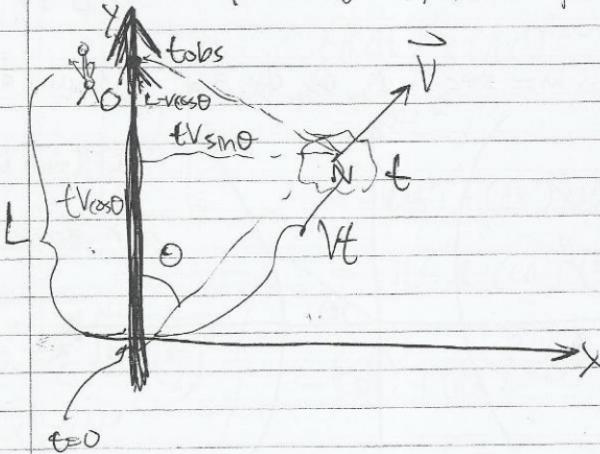
$$v = \frac{dx}{dt}$$



$\Delta s^2 > 0$ Χρησιμής Απέργης Επιφάνειας

$\Delta s^2 < 0$ Σπουδής Απέργης Επιφάνειας

$\Delta s^2 = 0$ προσέχεται Απέργης Επιφάνεια



$$c(t_{\text{obs}} - t) = \left[(L - Vt_{\text{rest}})^2 + (Vt_{\text{smo}})^2 \right]^{1/2}$$

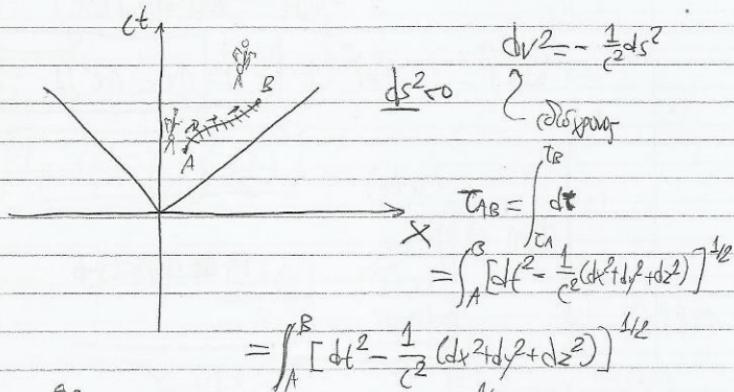
$$Vt \ll L$$

$$\Rightarrow t_{\text{obs}} = t \left(1 - \frac{V}{c} \cos \theta \right) + \frac{L}{c}$$

$$V_t = \frac{dx}{dt_{\text{obs}}} = \frac{dx}{dt} \frac{dt}{dt_{\text{obs}}} = \frac{V \sin \theta}{1 - \frac{V}{c} \cos \theta}, \text{ when } \theta \ll 1$$

real $V < c$

$$V \gg c$$

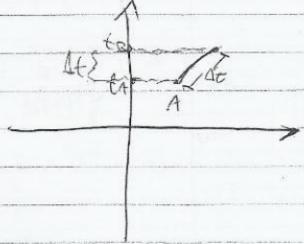


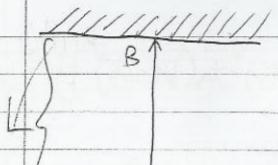
$$= \int_{t_A}^{t_B} dt \left[1 - \frac{1}{c^2} \left(\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 + \left(\frac{dz}{dt} \right)^2 \right) \right]^{1/2}$$

$$\Rightarrow \tau_{AB} = \int_{t_A}^{t_B} dt \sqrt{1 - \frac{V^2(t)}{c^2}} \quad \Rightarrow \tau_{AB} = |t_B - t_A|$$

$$V = \cos \theta \quad \Delta t = 4t \sqrt{1 - \frac{V^2}{c^2}}$$

$$d\tau = \sqrt{1 - \frac{V^2}{c^2}}$$





$$\Delta \tau = \frac{2L}{c} \quad \Delta t' = \frac{2}{c} \sqrt{L^2 + \left(\frac{\Delta x}{2}\right)^2}$$

$$\Delta x' = \sqrt{\Delta t'}$$

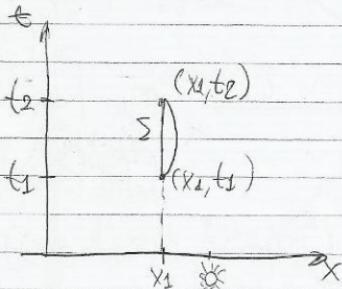
$$\Delta t' = \frac{2}{c} \left[L^2 + \left(\frac{\sqrt{\Delta t'}}{2} \right)^2 \right]^{1/2}$$

$$\Rightarrow \Delta t' = \frac{2}{c} \left[\left(\frac{\Delta x}{2} \right)^2 + \left(\frac{\sqrt{\Delta t'}}{2} \right)^2 \right]^{1/2}$$

$$\Rightarrow \Delta t'^2 = \frac{1}{c^2} \left[(\Delta x)^2 + (\sqrt{\Delta t'})^2 \right]$$

$$\Rightarrow \Delta t'^2 = (\Delta x)^2 + \left(\frac{v}{c} \right)^2 (\Delta t')^2$$

$$\Rightarrow (\Delta \tau)^2 = (\Delta t')^2 \left(1 - \frac{v^2}{c^2} \right) \Rightarrow \boxed{\Delta \tau = \Delta t' \sqrt{1 - \frac{v^2}{c^2}}}$$



$$\sum: t_2 - t_1$$

$$\Gamma: \Delta \tau < t_2 - t_1$$

Πρόσχ

1) Ο Γ είναι "αρχικός" σε $\frac{4}{5}c$ και επωρίζει σε όλο ομβρίο.

2) Ο Σ είναι τρέλαση βο γιαν. Πώς είναι γεράσει ο Γ;

$\Delta t < \infty$ xp.

$$2) \text{ ουαί } \sqrt{1 - \left(\frac{v}{c}\right)^2} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$$

από ότι γ δεν μπορεί να είναι πολύ μεγάλη

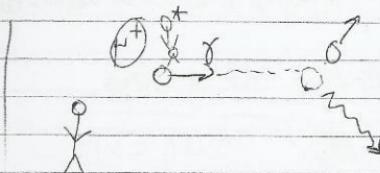
$$T = \gamma T_*$$

σ_{nw}

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$v \rightarrow 0 : \gamma \rightarrow 1$

$v \rightarrow c : \gamma \rightarrow \infty$



$$\tau_p^+(\gamma) = \gamma \tau_p^+(1)$$

διαφορά

$$\text{περιστατική: } \tau_p^+(1) - \frac{\tau_p^+(\gamma)}{\gamma}$$

$$(2 \pm 9) \cdot 10^{-4}$$

διαφορά

$$\frac{v}{c} = 0,9999\dots$$

$$\tau_p^+(1)$$

$$\Rightarrow \gamma \approx 29,3$$

$$\tau_p^+(\gamma) = (69,919 \pm 0,580) \mu s$$

X-πος Minkowski $\Rightarrow M_4 \cong \mathbb{R}^3 \times \mathbb{R}^+$

3 αριθμοί

$$ds'^2 \rightarrow ds^2$$

$$ct' = \cosh\theta \cdot (ct) - \sinh\theta \cdot (x) \quad y \leq y'$$

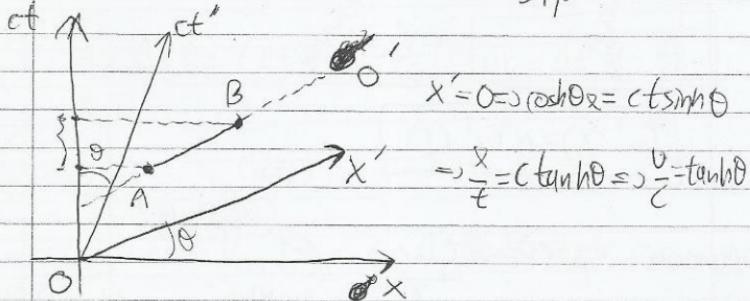
$$x' = \sinh\theta \cdot (ct) + \cosh\theta \cdot (x) \quad z = z' \quad \text{DE}(z_0 + \infty)$$

$$ds^2 = (cdt')^2 + (dx')^2$$

$$= -[(\cosh\theta(ct) - \sinh\theta(x))]^2 + [-\sinh\theta(ct) + \cosh\theta(x)]^2$$

$$= -(cdt)^2 + dx^2 \quad (\text{in } (dt)^2 + dx^2 \text{ even and odd})$$

Metric. Lorentz (boost) \rightarrow 3 dimensions



$$\cosh^2\theta - \sinh^2\theta = 1 \Rightarrow 1 = \frac{1}{\cosh^2\theta} + u^2$$

$$\Rightarrow \frac{1}{\cosh^2\theta} = 1 - u^2 \Rightarrow \cosh^2\theta = \frac{1}{1-u^2} \Rightarrow \cosh\theta = \frac{1}{\sqrt{1-u^2}}$$

$$\boxed{\cosh\theta}$$

$$\sinh^2 = \cosh^2 - 1 = \frac{1}{1-u^2} - 1 = \frac{1}{1-u^2} - \frac{1-u^2}{1-u^2} = \frac{2u^2}{1-u^2}$$

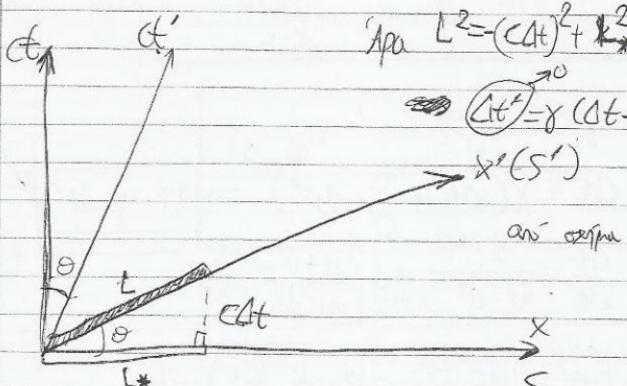
$$\Rightarrow \boxed{\sinh\theta = ux}$$

$$ct' = \gamma(ct) - \gamma u(x) \quad t' = \gamma(t - \frac{u}{c^2}x) \quad \text{Newton} \\ x' = -u\gamma(ct) + \gamma(x) \quad x' = \gamma(x - ut) \quad \stackrel{\text{def}}{=} \\ \Rightarrow t' = t \quad t = \gamma(t' + \frac{u}{c^2}x') \\ x' = x - ut \quad x = \gamma(x' + ut')$$

$$x_B - x_A = \Delta x = \gamma(4x' + u4t')$$

$$\Delta t = \gamma(4t' + \frac{u}{c^2}4x') \quad \text{or } \Delta t' = 0 \\ \Rightarrow \Delta x = \gamma 4x' \quad 4t = \gamma^2 \frac{u}{c^2} 4x'$$

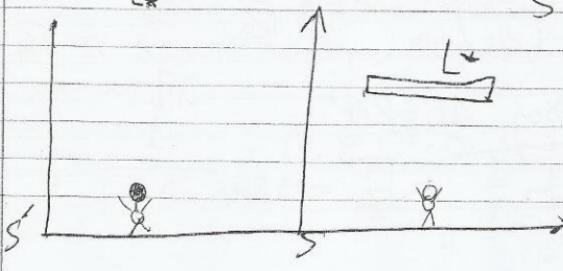
~~At \neq $\Delta t'$~~



$$\text{Spa } L^2 = (c\Delta t)^2 + L'^2$$

$$\Delta t' = \gamma(\Delta t - \frac{u}{c^2}4x)$$

analogous to $\Delta t' = 0$



$$\text{Divergencia}, \Delta t = \frac{v}{c} \Delta x \Rightarrow c \Delta t = \frac{v}{c} L$$

$$\text{Apa} L^2 = L_*^2 \left(1 - \frac{v^2}{c^2}\right) \Rightarrow L = L_* \sqrt{1 - \frac{v^2}{c^2}}$$

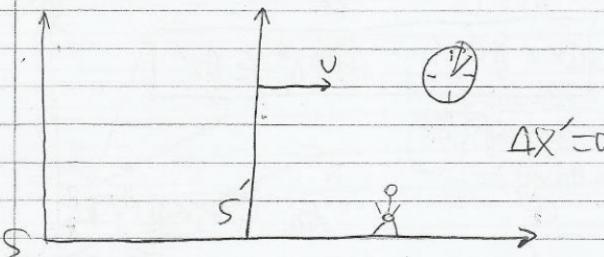
$$= L = \frac{1}{\gamma} L_*$$

$$T = \gamma T_* K$$

$$\frac{v}{c} = \beta$$

(Divergencia)

Επιτροπή, πολύ εύκαλπη για ταχύτητα πορειών που πρέπει να πρέπει
περιπότερο $\frac{1}{2}$ των πορειών ~~πάνω~~ ακίνητων
πορειών)



$$t = \gamma(t' + \frac{v}{c^2}x')$$

$$\Rightarrow \Delta t = \gamma(\Delta t' + \frac{v}{c^2} \Delta x) \Rightarrow \Delta t = \gamma \cdot \Delta t'$$

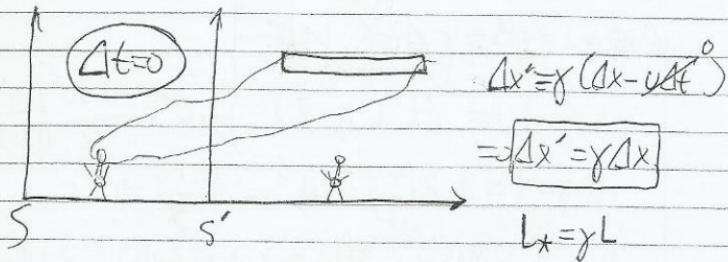
$$\Rightarrow \frac{\Delta t}{\Delta t'} = \frac{1}{\sqrt{1-\beta^2}} \Rightarrow \left(\frac{\Delta t}{\Delta t'}\right)^2 = \frac{1}{1-\beta^2}$$

$$\Rightarrow 1-\beta^2 = \left(\frac{\Delta t}{\Delta t'}\right)^2 \Rightarrow \beta = \sqrt{1 - \left(\frac{\Delta t}{\Delta t'}\right)^2} \quad \textcircled{1}$$

$$\text{Άρα ωντοτρογή } \Delta t = 2 \Delta t'$$

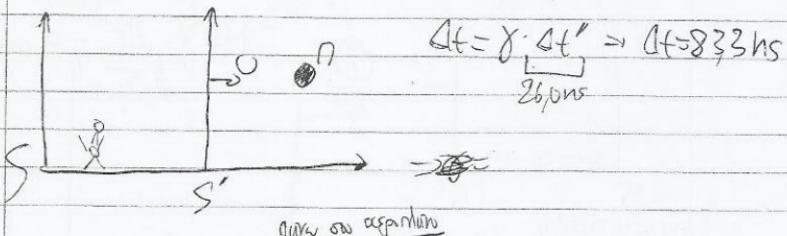
$$\textcircled{1} \Rightarrow \beta = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = 0,8666$$

• Ερώτηση: Με ποιά ταχύτηδα κινεται σήμερας
εσες μωρό από περιφέρεια της γης του νομού 50%



$$\theta = \sqrt{1 - \left(\frac{\Delta x}{\Delta x'}\right)^2} \quad \Delta x = \frac{1}{2} \Delta x' \Rightarrow \theta = \frac{\sqrt{3}}{2} = 0,866$$

• Ερώτηση: Ο πέρασ χρόνος γη-σύντη στον ουρανό είναι ακίνητος, είναι 26,0 hs.
Εγαν το πέρασ κινετή τη ταχ. $\theta = 0,95$, πώς είναι ο χρόνος σύντη στον ουρανό τη στιγμή που απορρίφεται από τη γη;



• Ερώτηση: Ενα ποδόστρεφα χρωνικό διάστημα 3600 s
δεν είναι το διάστημα κινέσαι με $v^2 = 900 \text{ m/s}$.
Σε τι χρωνικό διάστημα αλλοτριείται αυτό για παραπάνω
γιαν από γη;

$$\gamma = \frac{1}{\sqrt{1 - \theta^2}} = (1 - \theta^2)^{-1/2} \approx 1 + \frac{1}{2} \theta^2 + \dots$$

$$\Rightarrow \gamma - 1 \approx \frac{1}{2} \theta^2$$

$$\theta = \frac{900}{3 \cdot 10^8} \ll 1$$

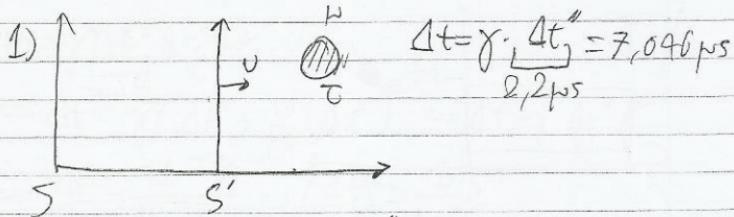
$$\begin{aligned}\Delta t - \Delta t' &= \gamma \Delta t' - \Delta t \\ &= \Delta t' (\gamma - 1) = \frac{\Delta t' \theta}{2} = \frac{36 \cdot 10^3}{2} \left(\frac{4 \cdot 10^8}{3 \cdot 10^8} \right)^2 \\ &= 1,8 \left(\frac{4}{3} \right)^2 \cdot 10^{-9} = \frac{16 \cdot 18}{9} \cdot 10^{-9} \text{ s}\end{aligned}$$

• Ερώτηση: Μείνω διαδοτάλ σύρριγμα με $N=N_0 \cdot e$

1) Ποιος είναι ο χρόνος γένησης του, οι ΑΣΑ που μετατρέπεται με $B=0,95$;

2) Πόσα έχουν μείνει από τα έξαν διαδοτάλ 3 km ;

$$(N_0 = N(t=0), \tau = 2,20 \text{ yrs})$$



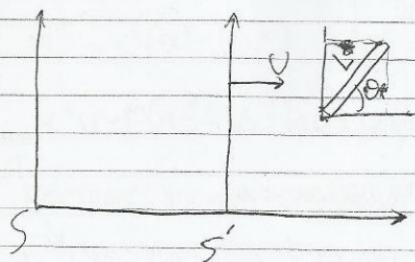
$$2) v = \frac{\Delta x'}{\Delta t'} \Rightarrow \Delta t' = \frac{\Delta x'}{v} \Rightarrow \frac{\Delta x'}{\frac{v}{c}} = \frac{\Delta x'}{\frac{v}{c}}$$

$$\Rightarrow \frac{\Delta x'}{\frac{v}{c}} = \frac{3 \cdot 10^3}{\frac{3 \cdot 10^8}{0,95}} = \frac{1}{10,53 \text{ yrs}} = \Delta t'$$

-10,53/22

$$\text{Α με } N = N_0 \cdot e = 0,225 N_0$$

• Aeronom



$$\Delta x' = L \cos \theta$$

$$\Delta y' = L \sin \theta$$

$$\Delta x' = \gamma \cdot \Delta x$$

$$\Delta y' = \gamma \cdot \Delta y$$

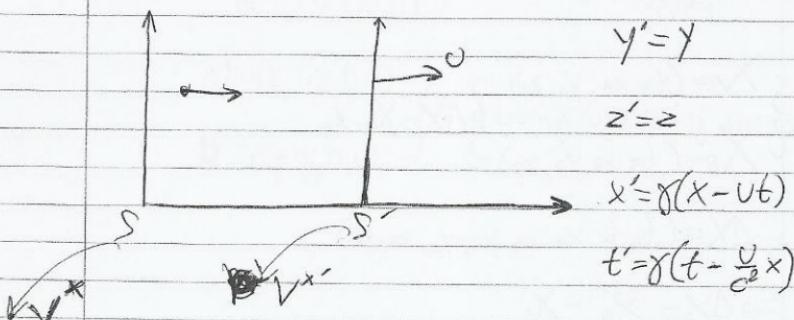
$$L^2 = (\Delta x)^2 + (\Delta y)^2 = \left(\frac{\Delta x'}{\gamma}\right)^2 + (\Delta y')^2$$

$$\Rightarrow L = \left[\frac{(\Delta x')^2}{\gamma^2} + (\Delta y')^2 \right]^{1/2} = \left[L^2 \cos^2 \theta + (L \sin \theta)^2 \right]^{1/2}$$

$$= \left[L^2 - L^2 \cos^2 \theta \right]^{1/2}$$

$$\Rightarrow L = L \sqrt{1 - \cos^2 \theta}$$

$$\tan \theta = \frac{\Delta y}{\Delta x} = \frac{\Delta y'}{\Delta x} \quad \gamma = (\tan \theta) \gamma$$



$$dx' = \gamma (dx - v dt)$$

$$dt' = \gamma (dt - \frac{v}{c^2} dx)$$

$$Vx' = \frac{dx'}{dt} = \frac{dx - vt}{dt - \frac{v}{c^2} dx} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2} \frac{dx}{dt}}$$

$$\Rightarrow V' = \frac{V - U}{1 - \frac{U}{c^2} V}$$

1) Η γενικότερη όρος $\epsilon \rightarrow \infty$: $V' = V - U$

αντίγραφη
Taylor

2) Σ πράγματα απλής περιστροφής.

$V' = (V - U)(1 - \frac{U}{c^2} V)^{-1} \approx (V - U)\left(1 + \frac{UV}{c^2} + O\left(\frac{UV}{c^2}\right)^2\right)$

3) $V = C$: $V' = \frac{C - U}{1 - \frac{U}{c^2}} = C$

Τετραδιώδευση

$$d = a^t e_t + a^x e_x + a^y e_y + a^z e_z = a^0 e_0 + a^1 e_1 + a^2 e_2 + a^3 e_3$$

$$= \sum_{i=0,1,2,3} a^i e_i = a^k e_k$$

$$X_A = (t_A, x_A, y_A, z_A)$$

$$X_B = (t_B, x_B, y_B, z_B)$$

$$\Delta X = X_B - X_A$$

$$\Delta X = (t_B - t_A, x_B - x_A, y_B - y_A, z_B - z_A)$$

$$\Rightarrow \Delta X = X_B^M - X_A^M$$

Aprówdomózg (Boosts)

$$\alpha^{t'} = \gamma(\alpha^t - v\alpha^x) \quad \alpha^{y'} = \alpha^y$$

(c=1)

$$\alpha^{x'} = \gamma(\alpha^x - v\alpha^t) \quad \alpha^{z'} = \alpha^z$$

Skalarny produkt $\alpha \beta$ w systemie Minkowskiego \rightarrow odc. LOS

$$\alpha \cdot \beta = \alpha^r e_r \cdot b^v e_v = \alpha^r b^r \underbrace{e_r e_v}_{\eta_{rv}} = \alpha^r b^v \underbrace{\eta_{rv}}_{\eta_{rv}}$$

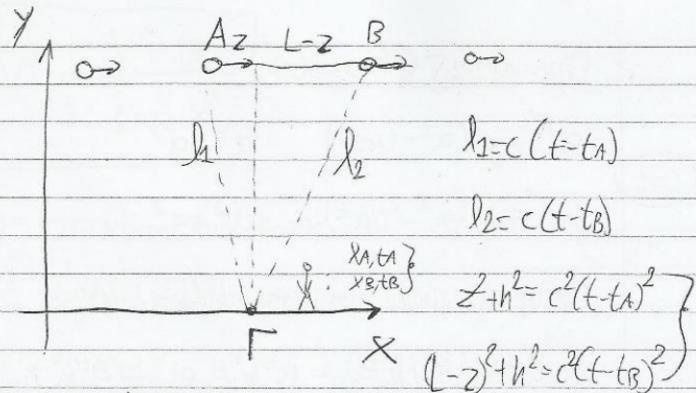
~~$\alpha^r b^v$~~

$$b_0 = b^r \eta_{rv} = b^0 \eta_{r0} + b^1 \eta_{r1} + b^2 \eta_{r2} + b^3 \eta_{r3} = -b^0$$

$$\alpha^r b_r = a^0 b_0 + a^1 b_1 + a^2 b_2 + a^3 b_3 = -a^0 b^0 + \vec{a} \cdot \vec{b} = -a^0 b^0 + \vec{a}' \cdot \vec{b}'$$

Przyk: $\begin{matrix} \alpha^r \\ \alpha^v \end{matrix} = \begin{pmatrix} 1, 0, 0, 0 \end{pmatrix}$ } $\Rightarrow \vec{a}' \cdot \vec{b}' = 0$
 $\begin{matrix} b^r \\ b^v \end{matrix} = \begin{pmatrix} 0, 1, 0, 0 \end{pmatrix}$

$$\begin{matrix} \alpha^r = (\gamma, v\gamma, 0, 0) \\ \alpha^v = (0, \gamma, 0, 0) \end{matrix} \quad \left\{ \Rightarrow \vec{a} \cdot \vec{b} = -\gamma(v\gamma) + (v\gamma)\gamma + 0 + 0 = 0 \right.$$



Translating along Γ at (x, t)

$$\sqrt{z^2 + h^2} = c(t - t_A)$$

$$\Rightarrow \sqrt{z^2 + h^2} = c(t - t_B)$$

$$\Rightarrow \sqrt{z^2 + h^2} = \sqrt{(L-z)^2 + h^2} = c(t_B - t_A)$$

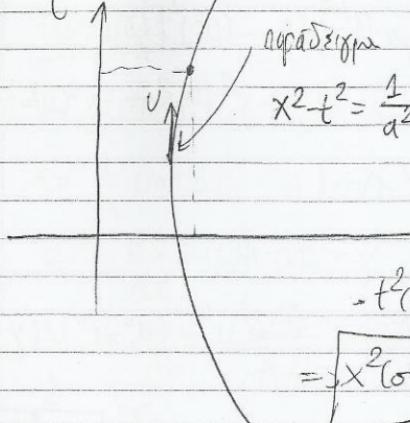
$$\Rightarrow \sqrt{z^2 + h^2} - \sqrt{(L-z)^2 + h^2} = c \left[\underbrace{\gamma(t_B - t_A)}_{\text{relative motion}} - \frac{v}{c} \underbrace{(x_B - x_A)}_{L^*} \right]$$

(equivalent expansion)

$$\text{for relative motion Lorentz: } \sqrt{z^2 + h^2} - \sqrt{(L^* - z)^2 + h^2} = -\frac{v}{c} L^*$$

координаты

t



координаты

$$x^2 - t^2 = \frac{1}{a^2}$$

$$t(\sigma) = \frac{1}{a} \sinh \sigma$$

$$x(\sigma) = \frac{1}{a} \cosh \sigma$$

$$\Rightarrow \sigma \in (-\infty, \infty)$$

$$-t^2(\sigma) + x^2(\sigma) = \frac{1}{a^2} (\cosh^2 \sigma - \sinh^2 \sigma) = \frac{1}{a^2}$$

$$= x^2(\sigma) - t^2(\sigma) = \frac{1}{a^2}$$

$$-ds^2 = dt^2$$

$$dt^2 = dt^2 - dx^2 = \left(\frac{1}{a} \cosh \sigma \cdot d\sigma \right)^2 - \left(\frac{1}{a} \sinh \sigma \cdot d\sigma \right)^2$$

$$\Rightarrow dt^2 = \frac{1}{a^2} (\cosh^2 \sigma - \sinh^2 \sigma) d\sigma^2$$

$$\Rightarrow d\tau = \frac{1}{a} d\sigma \Rightarrow \boxed{\tau = \frac{1}{a} \sigma}$$

$$t(\tau) = \frac{1}{a} \sinh(a\tau)$$

$$x(\tau) = \frac{1}{a} \cosh(a\tau)$$

q-координаты

$$\frac{dx(\tau)}{d\tau}$$

(τ)

$$\vec{U} = \frac{dx}{d\tau}$$

$$\frac{dx^\mu}{d\tau} = U^\mu$$

$$d\tau = dt \left(1 - \frac{U^2}{c^2} \right)^{-\frac{1}{2}}$$

$$U^\mu = \frac{dt}{d\tau} = \frac{1}{\sqrt{1 - \frac{U^2}{c^2}}}$$

$$\frac{dx}{d\tau} = \frac{dx}{dt} \cdot \frac{dt}{d\tau} = \frac{U}{\sqrt{1 - \frac{U^2}{c^2}}}$$

$$U^0 = \gamma$$

$$u \cdot u = u^p \cdot u_p = \eta_{\mu\nu} u^\mu u^\nu = -(u^0)^2 + (\vec{u})^2$$

$$= -\gamma^2 + \vec{u}^2 = \gamma^2(u^2 - 1)$$

$$\Rightarrow u \cdot u = \frac{1}{1-u^2} (u^2 - 1) = -1$$

$\sum_{\text{externale}} \rightarrow \text{Newton} \quad ? \rightarrow u^p = (u^0, \vec{u}) = (\gamma, \vec{\gamma} \vec{u})$

$\rightarrow \text{Maxwell} \checkmark$

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$v = \frac{\vec{u} \cdot \vec{c}}{c}$$

- $u^2 = u_p u^p = u_p u_\nu \eta^{\nu p} \text{ da } \eta^{\nu p} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$

$$= -(u^0)^2 + \vec{u} \cdot \vec{u}$$

$$= -(u^0)^2 + \vec{u} \cdot \vec{u}$$

$$\text{Apa } u^2 = -\gamma^2 + \gamma^2 \vec{u} \cdot \vec{u} = -1$$

Δυνατηγ (Newton)

$$1^{\text{os}} \text{ vpos Newton: } \frac{du}{dt} = 0 \quad (\text{σε } A \Sigma A \text{ δω } \vec{F} = 0)$$

$$2^{\text{os}} \text{ vpos Newton: } \frac{du}{dt} = f^p$$

τις μεταξ

Napăierea

$$x = \frac{1}{a} \cosh a\tau$$

$$\mathbf{a} \cdot \mathbf{u} = -(u^0)^2 + (u^x)^2 = -\cosh^2 a\tau + \sinh^2 a\tau$$

$$t = \frac{1}{a} \sinh a\tau$$

$$u^0 = \frac{dt}{d\tau} = \cosh a\tau = \gamma$$

$$v^x = \frac{dx}{dt} = \frac{dx/d\tau}{dt/d\tau} = \tanh a\tau$$

$$a^0 = \frac{du^0}{d\tau} = a \sinh a\tau$$

$$a^x = \frac{du^x}{d\tau} = a \coth h a\tau$$

$$p = m \cdot u \Rightarrow f = \frac{dp}{d\tau} \quad p \cdot p = m^2 \cdot u \cdot u = m^2$$

$$p^0 = mu^0 = m\gamma = \frac{m}{\sqrt{1-\beta^2}} \quad \vec{p} = m\gamma \vec{v} = \frac{m\vec{v}}{\sqrt{1-\beta^2}}$$

$$\text{re Taylor: } p^0 = m + \frac{1}{2} m \frac{v^2}{c^2} + \dots$$

$$\vec{p} = \left(m\vec{v} + \frac{1}{2} m \frac{v^2}{c^2} \vec{v}^x \right) + \dots$$

$$p^\mu = (m\gamma, m\vec{v}) = (E, \vec{p}) \Rightarrow p^2 = -m^2 \Rightarrow E^2 - \vec{p}^2 = m^2$$

$$\Rightarrow \boxed{E^2 = \vec{p}^2 + m^2}$$

Fizikai néprajz

$$\vec{V} = 0 \Rightarrow \vec{p} = 0 \quad E^2 = m^2 c^2 \Rightarrow E = m$$

Apa $E = mc^2$ Aivodaav
Einstein
 M_1 nücpa ✓

• H τερπομμή διατύπωση

$$p = (m\vec{v}, m\vec{V})$$

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \vec{f} = \frac{d\vec{p}}{dt} = \left(\frac{d\vec{p}}{dt} \right) \cdot \left(\frac{d\vec{t}}{d\tau} \right) = \gamma \vec{F}$$

$$\vec{f} \cdot \vec{u} = -f^0 \cdot u^0 + \vec{f} \cdot \vec{u} = 0$$

$$\Rightarrow f^0 = \frac{d\vec{p}^0}{dt} \Rightarrow \frac{dE}{dt} = \vec{F} \cdot \vec{V} = \gamma \vec{F} \cdot \vec{V} \Rightarrow f^0 = \frac{dp^0}{dt}$$

$$\Rightarrow \boxed{\frac{df}{dt} = \vec{F} \cdot \vec{V}} \Rightarrow \gamma \vec{F} \cdot \vec{V} = \frac{dp^0}{dt} \left(\frac{dt}{d\tau} \right)$$

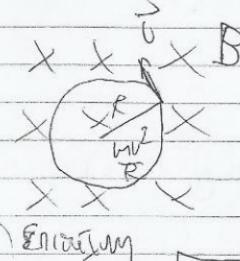
Abrony (Hibakunigas)

$$\vec{F} = q(\vec{V} \times \vec{B}) \Rightarrow f = q(u \otimes B)$$

Néprajzuk γ διαγ. Lorentz covarijanasajozése \vec{V}

Συμπίσθια ος β λεζάντα

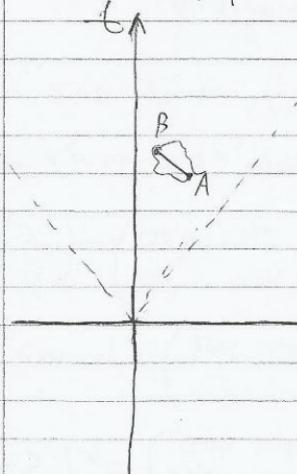
$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt} \left(\frac{m\vec{V}}{\sqrt{1-V^2}} \right) = \frac{1}{\sqrt{1-V^2}} m \frac{d\vec{V}}{dt} = m\vec{V} \left(\frac{d\vec{V}}{dt} \right)$$



$$\Rightarrow \frac{m\gamma V^2}{R} = qVB \Rightarrow R = \frac{mV\gamma}{qB} = \frac{|P|}{qB} = \frac{\sqrt{E^2 - m^2}}{qB}$$

$$\stackrel{(1)}{=} \gamma \vec{F} \cdot \vec{V} = 0 \quad (\vec{F} \perp \vec{V})$$

$$f^r = \gamma F^r \Rightarrow \gamma \cdot q \cdot B \cdot V \Rightarrow f^r = \frac{qB}{m} \sqrt{E^2 - m^2}$$



Καθε λεζάντη οως
επιπέδο x-t αναπτύχθει

$$\begin{aligned} \text{επιπέδο } AB \\ I_{AB} &= \int_A^B dt = \int_A^B \sqrt{dt^2 - dx^2 - dy^2 - dz^2} \\ &= \int_0^1 \sqrt{\left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2 + \left(\frac{dz}{ds}\right)^2} ds \end{aligned}$$