

Tέταρτη σειρά αντίδοσεων

Άσκηση 1:

$$h = \vec{e} \times \vec{n} \quad (1)$$

Τυχαίο πλανητικό διαδίκτυο. Από $n = \begin{pmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{pmatrix}$ ή $\vec{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$.

Μεταβολής σημειώσεων των \vec{e} και των \vec{n} με γνωμόνα \vec{h} :

$\vec{e}_x = [n_x \vec{e}_x] = h[n_x \vec{e}_x] = h[n_x \vec{e}_x]$

$\vec{e}_y = [n_y \vec{e}_y] = h[n_y \vec{e}_y] = h[n_y \vec{e}_y]$

$\vec{e}_z = [n_z \vec{e}_z] = h[n_z \vec{e}_z] = h[n_z \vec{e}_z]$

$$n \cdot h = (n_x x + n_y y + n_z z) \cdot (y p_{xz} - y p_y) \vec{x} + (x p_{yz} - x p_x) \vec{y} + (x p_{xy} - y p_x) \vec{z} =$$

$$= -i \hbar \left[n_x \left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right) + n_y \left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right) + n_z \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) \right]$$

$$n \cdot h, x \vec{x} = x^2 \vec{x} - x \vec{n} \cdot \vec{h} \stackrel{(3)}{=} x^2 - i \hbar \left[n_x \left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right) \right] x +$$

$$+ n_y \left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right) x + p_{xz} \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) \vec{x} + i \hbar \left[x n_x \frac{\partial}{\partial z} - \right.$$

$$\left. - x \frac{\partial}{\partial y} \right] + x n_y \left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right) + x n_z \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) \vec{x} =$$

$$= -i \hbar x \left[n_x y \frac{\partial}{\partial z} - n_x z \frac{\partial}{\partial y} + n_y x \frac{\partial}{\partial z} - n_y z \frac{\partial}{\partial x} + n_z x \frac{\partial}{\partial y} - n_z y \frac{\partial}{\partial x} \right] -$$

$$- n_y x \frac{\partial}{\partial z} + n_x x \frac{\partial}{\partial z} - n_z y \frac{\partial}{\partial z} \left(x \vec{x} \right) - n_x y \frac{\partial}{\partial z} + n_x z \frac{\partial}{\partial y} -$$

$$= n_y x z \frac{\partial}{\partial x} + n_y x z \frac{\partial}{\partial z} - n_x x z \frac{\partial}{\partial y} + n_z x y \frac{\partial}{\partial x} =$$

$$= -i \hbar x \left[n_y x z \frac{\partial}{\partial x} - n_y x z \frac{\partial}{\partial z} - n_x x z \frac{\partial}{\partial y} + n_z x y \frac{\partial}{\partial x} \right]$$

$$= -i \hbar x \left[n_y x z \frac{\partial}{\partial x} + n_y z x \frac{\partial}{\partial x} - n_y x z \frac{\partial}{\partial x} - n_x y \frac{\partial}{\partial x} - n_x y \frac{\partial}{\partial x} \right]$$

$$- n_x y \frac{\partial}{\partial x} \right] = i \hbar (n_y z - n_y z) \vec{x} \Rightarrow [n \cdot h, x \vec{x}] = -i \hbar (n_y z - n_y z)$$

Ορισμός λόγω μεταβολής σημειώσεων βρίσκεται:

$$- n_y y \frac{\partial}{\partial z} + i \hbar (n_x z - n_x z) \quad (5)$$

$$- n_x z \frac{\partial}{\partial y} = 2i \hbar (n_x y - n_y x) \quad (6)$$

$$n \cdot h, y \vec{y} = x^2 \vec{y} - p_{xz} \vec{y} \stackrel{(3)}{=} x^2 - i \hbar \left[n_y \left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right) \right] (-i \hbar) \vec{y}$$

$$- n_y \left(z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right) (-i \hbar) \vec{y} + n_z \left(x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right) (-i \hbar) \vec{y} - (-i \hbar) \vec{y}$$

$$\begin{aligned}
 & b) \left[n_x \left(y \frac{\partial \psi}{\partial z} - z \frac{\partial \psi}{\partial y} \right) + n_y \left(z \frac{\partial \psi}{\partial x} - x \frac{\partial \psi}{\partial z} \right) + n_z \left(x \frac{\partial \psi}{\partial y} - y \frac{\partial \psi}{\partial x} \right) \right] = \\
 & = x^2 (-h^2) \left[n_x y \frac{\partial^2 \psi}{\partial z^2} - n_x z \frac{\partial^2 \psi}{\partial y \partial z} + n_y z \frac{\partial^2 \psi}{\partial x^2} - n_y x \frac{\partial^2 \psi}{\partial z \partial x} + \right. \\
 & \quad \left. + n_z x \frac{\partial^2 \psi}{\partial y \partial z} - n_z y \frac{\partial^2 \psi}{\partial x^2} - n_x w \frac{\partial^2 \psi}{\partial z^2} + n_x z \frac{\partial^2 \psi}{\partial x \partial z} - n_y z \frac{\partial^2 \psi}{\partial y \partial z} \right. \\
 & \quad \left. + n_y x \frac{\partial^2 \psi}{\partial z \partial x} \right] - n_z \frac{\partial}{\partial x} \left(x \frac{\partial \psi}{\partial y} \right) + n_y \frac{\partial}{\partial y} \left(x \frac{\partial \psi}{\partial y} \right) - \\
 & = -x h^2 \left[-n_y x \frac{\partial^2 \psi}{\partial z \partial x} + n_y \frac{\partial^2 \psi}{\partial x^2} + n_y x \frac{\partial^2 \psi}{\partial z \partial x} + n_x z \frac{\partial^2 \psi}{\partial y \partial z} \right. \\
 & \quad \left. - n_z \frac{\partial x}{\partial x} \frac{\partial \psi}{\partial y} - n_z x \frac{\partial \psi}{\partial y} \right] = x^2 (-h^2) \left[n_y \frac{\partial^2 \psi}{\partial z^2} - n_z \frac{\partial^2 \psi}{\partial y \partial z} \right] \psi = \\
 & \text{in L, } n_x \psi \rightarrow x(-h^2) \left[n_y \frac{\partial^2 \psi}{\partial z^2} - n_z \frac{\partial^2 \psi}{\partial y \partial z} \right]
 \end{aligned}$$

Opoisys, kaiu kiekliukuij užteikėsi už būtų kaip:

$$[n \cdot h, n_y \psi] = i^2 (-h^2) \left[n_z \frac{\partial^2 \psi}{\partial x^2} - n_x \frac{\partial^2 \psi}{\partial z^2} \right] \quad (8)$$

$$[n \cdot h, n_z \psi] = i^2 (-h^2) \left[n_x \frac{\partial^2 \psi}{\partial y^2} - n_y \frac{\partial^2 \psi}{\partial x^2} \right] \quad (9)$$

$$\begin{aligned}
 & [n \cdot h, n_y \psi] \stackrel{(4)(5)(6)}{=} -i^2 h (n_y \psi - n_z \psi) - i^2 h (n_z x - n_x z) - 2i^2 h (n_y - n_x) \psi = \\
 & = -i^2 h (n_y \psi + n_z \psi) - i^2 h (n_z x - n_x z) - 2i^2 h (n_y - n_x) \psi = \\
 & = -i^2 h \left[n_y \psi + n_z \psi + n_z x - n_x z + n_x \psi + n_z \psi \right] = -i^2 h n_x \psi = \\
 & [n \cdot h, n_z \psi] \stackrel{(8)(9)}{=} [n_y \psi, n_x \psi + n_y \psi + n_z \psi] = [n \cdot h, n_y \psi] + [n \cdot h, n_z \psi] = \\
 & = -i^2 h \left[n_y \frac{\partial^2 \psi}{\partial z^2} - n_x \frac{\partial^2 \psi}{\partial y \partial z} + i^2 (-h) \right] \left[n_z \frac{\partial^2 \psi}{\partial x^2} - n_x \frac{\partial^2 \psi}{\partial z \partial x} + i^2 (-h) \right] \left[n_x \frac{\partial^2 \psi}{\partial y^2} - n_y \frac{\partial^2 \psi}{\partial x^2} \right] \quad (10) \\
 & - n_y \frac{\partial^2 \psi}{\partial z^2} = -i^2 h \left[x \left(n_y (-i^2 h) \frac{\partial^2 \psi}{\partial z^2} - n_x (-i^2 h) \frac{\partial^2 \psi}{\partial y \partial z} \right) + y \left(n_z (-i^2 h) \frac{\partial^2 \psi}{\partial x^2} - n_x (-i^2 h) \frac{\partial^2 \psi}{\partial z \partial x} \right) \right] = -i^2 h \left[x (n_y n_z - n_x n_y) \right] \quad (11) \\
 & - n_x \frac{\partial^2 \psi}{\partial z \partial x} = -i^2 h \left[x \left(n_x (-i^2 h) \frac{\partial^2 \psi}{\partial y \partial z} - n_y (-i^2 h) \frac{\partial^2 \psi}{\partial x^2} \right) + z \left(n_z (-i^2 h) \frac{\partial^2 \psi}{\partial z \partial x} - n_y (-i^2 h) \frac{\partial^2 \psi}{\partial x^2} \right) \right] = -i^2 h \left[x (n_z n_y - n_x n_z) \right] \quad (12)
 \end{aligned}$$

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Στατικός Αριθμός

$$\begin{aligned}
 & [n \cdot L \xrightarrow{\rightarrow} \vec{n}] = [n \cdot L \xrightarrow{\rightarrow} \vec{n} + \vec{n}] \xrightarrow{\rightarrow} \vec{n} \stackrel{(11)}{=} -i \vec{k} (\vec{n} \times \vec{n}) \cdot \vec{n} + \vec{k} (\vec{n} \times \vec{n}) = \\
 & = -i \vec{k} (\vec{n} \times \vec{n}) + \vec{k} \cdot (\vec{n} \times \vec{n}) \stackrel{(12)}{=} -i \vec{k} \vec{n} / (\vec{n} \times \vec{n}) = -i \vec{k} n \cdot \vec{n} \\
 & \stackrel{(13)}{=} [n \cdot L \xrightarrow{\rightarrow} \vec{n}] = [n \cdot L \xrightarrow{\rightarrow} \vec{n} + \vec{z}] \xrightarrow{\rightarrow} \vec{n} \stackrel{(10)(11)}{=} \vec{k} (\vec{n} \times \vec{z}) \cdot \vec{n} + \vec{z} \cdot (\vec{n} \times \vec{n}) = \\
 & = -i \vec{k} [\vec{n} \cdot (\vec{n} \times \vec{z}) + \vec{z} \cdot (\vec{n} \times \vec{n})] = i \vec{k} n \vec{z}
 \end{aligned}$$

Αρχήν 2:

$$K(\theta, \phi) = N(Y_{i-1} + Y_{i,0} + 2Y_{i,1}) \text{ κανονικοποιείται (1)}$$

\Rightarrow Η μη συγκριτική απόφασης

$$\begin{aligned}
 & \text{a) } N = \frac{1}{\sqrt{6}} \stackrel{(1)}{\Rightarrow} N(Y_{i-1} + Y_{i,0} + 2Y_{i,1}) | N(Y_{i-1} + Y_{i,0} + 2Y_{i,1}) \geq 1 \Rightarrow \\
 & \Rightarrow N(Y_{i-1} + Y_{i,0} + 2Y_{i,1}) \leq N(Y_{i-1}) + N(Y_{i,0}) + N(2Y_{i,1}) \geq 1 \Rightarrow \\
 & \Rightarrow N^2 [1 + 1 + 4 \cdot 1] = 6 | N^2 = 1
 \end{aligned}$$

$$N = \frac{1}{\sqrt{6}} \quad (2)$$

$$(b) \Delta L_x = j$$

$$\Delta(L_x^2) = j$$

$$(1) \stackrel{(2)}{\Rightarrow} \psi(0, \phi) = \frac{1}{\sqrt{6}} (Y_{i-1} + Y_{i,0} + 2Y_{i,1}) \quad (3)$$

$$\langle L_x^2 \rangle = \langle \psi | L_x \cdot L_x | \psi \rangle$$

$$\text{Όμως } L_x^2 | \psi \rangle \stackrel{(3)}{=} L_x \cdot L_x | \frac{1}{\sqrt{6}} (Y_{i-1} + Y_{i,0} + 2Y_{i,1}) \rangle = \frac{1}{\sqrt{6}} L_x | L_x | Y_{i-1} \rangle +$$

$$L_x | Y_{i,0} \rangle + 2 L_x | Y_{i,1} \rangle = \frac{1}{\sqrt{6}} L_x [-1 \vec{k} | Y_{i-1} \rangle + 0 \vec{k} | Y_{i,0} \rangle + 2 \cdot 1 \vec{k} | Y_{i,1} \rangle]$$

$$= \frac{1}{\sqrt{6}} [-\vec{k} L_x | Y_{i-1} \rangle + 2 \vec{k} L_x | Y_{i,1} \rangle] = \frac{1}{\sqrt{6}} [-\vec{k} (-\vec{k}) | Y_{i-1} \rangle + 2 \vec{k} \cdot \vec{k} | Y_{i,1} \rangle]$$

$$= \frac{1}{\sqrt{6}} [\vec{k}^2 | Y_{i-1} \rangle + 2 \vec{k}^2 | Y_{i,1} \rangle]$$

$$\text{Συντετρούσσεις } \langle L_x^2 \rangle = \frac{1}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}} [\vec{k}^2 \cdot 1 \cdot 1 \langle Y_{i-1} | Y_{i-1} \rangle + \vec{k}^2 \cdot 2 \langle Y_{i,1} | Y_{i,1} \rangle]$$

$$= \frac{1}{6} [\vec{k}^2 \cdot 1 + \vec{k}^2 \cdot 4 \cdot 1] = \frac{5 \vec{k}^2}{6} \quad (4)$$

$$\Delta \text{ στατικός } \langle L_x \rangle = \langle \psi | L_x | \psi \rangle$$

$$\Delta \text{ στατικός } L_x / \sqrt{6} = \frac{1}{\sqrt{6}} [-\vec{k} | Y_{i-1} \rangle + 2 \vec{k} | Y_{i,1} \rangle]$$

$$\begin{aligned}
 \text{Apr} \langle h_2 \rangle &= \frac{1}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}} [16k \langle y_{i-1} | y_{i-1} \rangle + 2 \cdot 2k \langle y_{i+1} | y_{i+1} \rangle] = \frac{4k \cdot 1 \cdot k \cdot 1}{6} \\
 &= \frac{3k}{2} = \frac{k}{2} (5) \\
 \langle L_2 \rangle^2 &= \langle h_2 \rangle \langle h_2 \rangle \stackrel{(4)(5)}{=} \frac{5k^2}{6} - \left(\frac{k}{2}\right)^2 = \frac{5k^2}{6} - \frac{k^2}{4} = \frac{10k^2 - 3k^2}{12} \\
 \cancel{\frac{7k}{12}} \Rightarrow \langle L_2 \rangle &= \sqrt{\frac{7}{12}} k (6) \\
 \langle L^2 \rangle &\geq \langle 4 | L^2 | L^2 | \cancel{\frac{7}{12}} \rangle \\
 \text{Opus } L^2 | L^2 | \cancel{\frac{7}{12}} &\stackrel{(3)}{=} L^2 | L^2 | \frac{1}{\sqrt{6}} (y_{i-1} + y_{i+1} + 2y_{i+1}) \rangle = \\
 &= \frac{1}{\sqrt{6}} L^2 [L^2 | y_{i-1} \rangle + L^2 | y_{i+1} \rangle + 2L^2 | y_{i+1} \rangle] = \\
 &= \frac{1}{\sqrt{6}} L^2 [k^2(1+k) | y_{i-1} \rangle + k^2(1+k) | y_{i+1} \rangle + 2k^2(1+k) | y_{i+1} \rangle] = \\
 &= \frac{1}{\sqrt{6}} [2k^2 | y_{i-1} \rangle + 2k^2 | y_{i+1} \rangle + 4k^2 | y_{i+1} \rangle] - \frac{1}{\sqrt{6}} [2k^2 | L^2 | y_{i-1} \rangle + 2k^2 | L^2 | y_{i+1} \rangle] \\
 &= 4k^2 L^2 | y_{i+1} \rangle = \frac{1}{\sqrt{6}} [2k^2 \cdot 2k^2 | y_{i-1} \rangle + 2k^2 \cdot 2k^2 | y_{i+1} \rangle + 4k^2 \cdot 2k^2 | y_{i+1} \rangle] \\
 &= \frac{1}{\sqrt{6}} [4k^4 | y_{i-1} \rangle + 4k^4 | y_{i+1} \rangle + 8k^4 | y_{i+1} \rangle] \\
 \text{Apr} \langle \vec{L}^4 \rangle &= \frac{1}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}} [1 \cdot 4k^4 \langle y_{i-1} | y_{i-1} \rangle + 1 \cdot 4k^4 \langle y_{i+1} | y_{i+1} \rangle + 2 \cdot 8k^4 y_{i+1} | y_{i+1} \rangle \\
 &= \frac{4k^4 \cdot 1 + 4k^4 \cdot 1 + 16k^4 \cdot 1 - 24k^4}{6} = 4k^4 (7) \\
 \langle \vec{L}^2 \rangle &\geq \langle 4 | L^2 | \cancel{\frac{7}{12}} \rangle \\
 \text{A.o. } L^2 | \cancel{\frac{7}{12}} &\geq \frac{1}{\sqrt{6}} [2k^2 | y_{i-1} \rangle + 2k^2 | y_{i+1} \rangle + 4k^2 | y_{i+1} \rangle] \\
 \text{Apr} \langle \vec{L}^2 \rangle &= \frac{1}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}} [2k^2 \cdot 1 \langle y_{i-1} | y_{i-1} \rangle + 2k^2 \cdot 1 \langle y_{i+1} | y_{i+1} \rangle + 4k^2 \cdot 2 \langle y_{i+1} | y_{i+1} \rangle] \\
 &= \frac{2k^2 \cdot 1 + 2k^2 \cdot 1 + 8k^2 \cdot 1}{6} = \frac{12k^2}{6} = 2k^2 (8) \\
 \langle \vec{L}^2 \rangle^2 &\leq \langle \vec{L}^2 \rangle - \langle \vec{L}^2 \rangle^2 \stackrel{(7)(8)}{=} 4k^4 - (2k^2)^2 = 4k^4 - 4k^4 = 0 \Rightarrow \langle \vec{L}^2 \rangle =
 \end{aligned}$$

Σταθμικ Ασένος

Άρχοντας 3:

$$\begin{aligned}
 & \text{(a) Συμμετοχή με κυματογενάρηση } \psi(\zeta, \theta, \varphi) = g(\zeta)(\sin \theta \sin \varphi i + \sin \theta \cos \varphi j + (\cos \theta)k \\
 & \text{V. d. το συμμετοχό είναι καταπολεμητικό από την πλεοναρχία όπου υπάρχει} \\
 & \omega(\zeta, \theta, \varphi) = (-i\hbar) - \sin \theta \frac{\partial}{\partial \theta} - \frac{\cos \theta}{\sin \theta} \frac{\partial}{\partial \varphi} + (\zeta, \theta, \varphi) \\
 & = i\hbar \sin \theta \frac{\partial g(\zeta)}{\partial \theta} (\sin \theta \sin \varphi i + \cos \theta j) - i\hbar \frac{\cos \theta}{\sin \theta} \frac{\partial g(\zeta)}{\partial \varphi} (\sin \theta \sin \varphi i + \cos \theta j) \\
 & , \frac{\partial^2 g(\zeta)}{\partial \theta^2} (\sin \theta \sin \varphi i + \cos \theta j) = i\hbar \sin \theta g(\zeta) (\cos \theta \sin \theta - \sin \theta) + \\
 & + i\hbar \frac{\cos \theta}{\sin \theta} \cos \theta \cdot g(\zeta) \sin \theta = i\hbar g(\zeta) \cos \theta \sin^2 \theta + i\hbar \sin \theta g(\zeta) \sin \theta \sin \varphi + \\
 & + i\hbar \frac{\cos \theta}{\sin \theta} \cos \theta \cos \theta = i\hbar g(\zeta) \cos \theta \sin^2 \theta + i\hbar g(\zeta) \sin \theta \sin \varphi \\
 & = Rg(\zeta) (\sin \theta \sin \varphi i + \cos \theta j) = R\psi(\zeta, \theta, \varphi) \Rightarrow \hbar = R(2) \rightarrow \text{καταπολεμητικό} \\
 & \text{μεταβολή}
 \end{aligned}$$

Τι διαβέβαιο μέρης έχει ο χαρακτής της καταπολεμητικότητας;

Τι διανομή της προκύπτει η καταπολεμητικότητα;

Καταπολεμητικότητα;

$$x=0 \Rightarrow \zeta \sin \theta \cos \theta = 0 \quad \forall \zeta, \theta, \varphi \Rightarrow \sin \theta = 0 \quad (3)$$

$$\begin{aligned}
 & \psi(i\hbar \varphi) = -i\hbar \frac{\partial}{\partial \varphi} g(\zeta) (\sin \theta \sin \varphi i + \cos \theta j) = -i\hbar g(\zeta) \sin \theta \cos \varphi j - i\hbar g(\zeta) i \\
 & = 0 \quad \forall \zeta, \theta, \varphi \Rightarrow \hbar = 0 \rightarrow 1 \text{ μόνη μεταβολή μεταβολής} \\
 & \text{μεταβολής} \Rightarrow \text{μεταβολή μεταβολής} \Rightarrow \text{μεταβολή μεταβολής}
 \end{aligned}$$

$\Rightarrow \hbar = 1$

Άρχοντας 4:

Συγκριτική κανονισμού

$$\Psi = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad (1)$$

$$| \Psi(t=0) \rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad (2)$$

$$\begin{aligned}
 & \dot{\Psi}(t=0) = j \\
 & \dot{\Psi}(t \neq 0) = e^{-i\hbar t/R} | \Psi(t=0) \rangle \quad (3)
 \end{aligned}$$

Θεωρούμε $\mathbb{C}^{iAt/k} = e^A \Rightarrow A - \frac{i}{k} H \stackrel{(1)}{=} -\frac{i}{k} \begin{bmatrix} a & b \\ b^* & a \end{bmatrix}$

Άρα $e^{-iAt/k} = e^A \cong I + A + \frac{A^2}{2!} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{i}{k} \begin{bmatrix} a & b \\ b^* & a \end{bmatrix} + \frac{1}{2!} \left(\frac{i}{k} \right)^2 \begin{bmatrix} a^2 + b^2 & ab \\ ab^* & a^2 + b^2 \end{bmatrix} =$

$$= \left(\begin{array}{cc} 1 - \frac{i}{k} a - \frac{t^2(a^2+b^2)}{2k^2} & \frac{-i}{k} b - \frac{t^2 ab}{2k^2} \\ \frac{-i}{k} b^* - \frac{t^2 ab^*}{2k^2} & 1 - \frac{i}{k} a - \frac{t^2(a^2+b^2)}{2k^2} \end{array} \right) \quad (4)$$

$\stackrel{(2)(4)}{\Rightarrow} H(+0) \Rightarrow \begin{pmatrix} 1 - \frac{i}{k} a - \frac{t^2(a^2+b^2)}{2k^2} & -\frac{i}{k} b - \frac{t^2 ab}{2k^2} \\ -\frac{i}{k} b^* - \frac{t^2 ab^*}{2k^2} & 1 - \frac{i}{k} a - \frac{t^2(a^2+b^2)}{2k^2} \end{pmatrix}$

$$= \begin{pmatrix} -\frac{t^2 ab}{k^2} - \frac{itb}{k} \\ 1 - \frac{t^2(a^2+b^2)}{2k^2} - \frac{ita}{k} \end{pmatrix}$$

Άσκηση 5:

Κυματοειδήση των ΟΠΤV από τη λειτουργία: $\psi = 3c x_+ + c x_-$ (1)

x_+, x_- διατίθενται στην παραγράφη της

$$\text{Σ } x_+ - \frac{k}{2} x_+ \Rightarrow x_+ = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2)$$

$$\text{Σ } x_- - \frac{k}{2} x_- \Rightarrow x_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (3)$$

Συστήματα καραβιών (ηρεμίας)

$$(1) \stackrel{(2)(3)}{\Rightarrow} \psi = 3c \begin{pmatrix} 0 \\ 1 \end{pmatrix} + c \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3c \\ c \end{pmatrix}$$

$$\psi \psi^* = 1 \Rightarrow (3c^* c) \begin{pmatrix} 3c \\ c \end{pmatrix} = 1 \Rightarrow 9|c|^2 + |c|^2 = 1 \Rightarrow |c|^2 = \frac{1}{10} \Rightarrow c = \frac{1}{\sqrt{10}}$$

Άρα $\psi = \frac{1}{\sqrt{10}} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ (4)

Μετρήση το ΟΠΤV των ομηραδίων στην κατεύθυνση ενός αγαντ

$$S_n = \frac{1}{\sqrt{3}} (\sqrt{2} S_x + S_y) \quad (5)$$

Σταθμική Ανέρευτη

(a) Τύπος γρα τη στην της σχηματίδιου σε αυτή την καρτεσιανή:

$$S_n = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2} S_x + S_y \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{30}} \begin{pmatrix} \sqrt{2} \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{30}} \begin{pmatrix} \sqrt{2} \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$\begin{pmatrix} \sqrt{2} \\ 2 \end{pmatrix} = \frac{\sqrt{2}}{\sqrt{30}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{\sqrt{2}}{\sqrt{3}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$= \frac{\sqrt{2}}{2\sqrt{3}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{\sqrt{2}}{2\sqrt{3}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$\vec{n} = x \sin \theta \cos \phi + y \sin \theta \sin \phi + z \cos \theta \cos \phi$$

$$S_n = S_n \cdot \vec{n} = \sin \theta \cos \phi S_x + \sin \theta \sin \phi S_y + \cos \theta \cos \phi$$

$$\left\{ \begin{array}{l} \sin \theta \cos \phi = \frac{\sqrt{2}}{\sqrt{3}} \\ \sin \theta \sin \phi = \frac{1}{\sqrt{3}} \\ \cos \theta = 0 \end{array} \right. \Rightarrow \sin \theta = -1 \Rightarrow \sin \theta = -\frac{1}{\sqrt{3}}, \cos \theta = -\frac{\sqrt{2}}{\sqrt{3}}$$

$$\left\{ \begin{array}{l} \sin \theta \cos \phi = \frac{\sqrt{2}}{\sqrt{3}} \\ \sin \theta \sin \phi = \frac{1}{\sqrt{3}} \\ \cos \theta = 0 \end{array} \right. \Rightarrow \sin \theta = \pm 1 \Rightarrow$$

Άστο την (6) βρίσκουμε τις λύσεις των S_n : $\det |S_n - \lambda I| = 0 \Rightarrow$

$$\begin{vmatrix} -\lambda & \frac{\sqrt{2}}{\sqrt{3}}(\sqrt{2}-i) \\ \frac{\sqrt{2}}{\sqrt{3}}(\sqrt{2}+i) & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - \frac{\sqrt{2}^2}{4\sqrt{3}}(\sqrt{2}-i)(\sqrt{2}+i) = 0 \Rightarrow$$

$$\lambda^2 - \frac{\sqrt{2}^2(2i)}{3 \cdot 4} = 0 \Rightarrow \lambda^2 = \frac{\sqrt{2}^2(2i)}{3 \cdot 4} \Rightarrow \lambda = \pm \frac{\sqrt{2}}{2} = \pm \frac{\sqrt{2}}{2}$$

$$S_n = \pm \frac{\sqrt{2}}{2} (g)$$

(b) Άνοιξες κυριαρχητικούς:

$$\begin{vmatrix} \frac{\pm \sqrt{2}}{2} & \frac{\sqrt{2}}{\sqrt{3}}(\sqrt{2}-i) \\ \frac{\sqrt{2}}{\sqrt{3}}(\sqrt{2}+i) & \frac{\pm \sqrt{2}}{2} \end{vmatrix} (g) = 0 \Rightarrow \begin{cases} \frac{\pm \sqrt{2}}{2} \left(\pm 1_a + \frac{\sqrt{2}-i}{\sqrt{3}} b \right) = 0 \\ \frac{\sqrt{2}}{\sqrt{3}} \left(\frac{\sqrt{2}+i}{\sqrt{3}} a \pm 1_b \right) = 0 \end{cases} \Rightarrow$$

$$\begin{cases} \pm a + \frac{\sqrt{2}-i}{\sqrt{3}} b = 0 \\ \frac{\sqrt{2}+i}{\sqrt{3}} a \pm b = 0 \end{cases} \Rightarrow a = \pm \frac{\sqrt{2}-i}{\sqrt{3}} b$$

2

Остане да сме $|a|^2 + |b|^2 = 1 \Rightarrow$
 ~~$\frac{1}{\sqrt{2}} - i$~~ $b * \frac{\sqrt{2} + i}{\sqrt{3}}$ $+ |b|^2 = 1 \Rightarrow \frac{2+i}{3} |b|^2 + |b|^2 = 1 \Rightarrow 2|b|^2 = 1 \Rightarrow$

$$b = \frac{1}{\sqrt{2}}$$

т.к. $a = \pm \frac{\sqrt{2}-i}{\sqrt{3}} \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}-i}{\sqrt{6}}$. Ако:

$$\begin{cases} S_n = -\frac{k}{2} \\ S_n = +\frac{k}{2} \end{cases} \Rightarrow \begin{cases} x_-^{(n)} = \left(\frac{\sqrt{2}-i}{\sqrt{6}} \right) \\ x_+^{(n)} = \left(-\frac{\sqrt{2}-i}{\sqrt{6}} \right) \end{cases} \quad (9)$$

(*) Ето че, P_- е η стапорача на вгради $x_-^{(n)}$ каде P_+ е η стапорача на вгради $x_+^{(n)}$.

\Rightarrow

$$x \pm = \frac{x_-^{(n)}}{1 + \frac{1}{P_-}} \stackrel{(4), (8)}{=} \frac{\frac{1}{\sqrt{6}} \left(\frac{\sqrt{2}-i}{\sqrt{2}} \right)}{1 + \frac{3\sqrt{2}-3i}{\sqrt{6}}} = \frac{\frac{1}{\sqrt{6}} \left(\frac{\sqrt{2}-i}{\sqrt{2}} \right)}{\frac{\sqrt{6} + 3\sqrt{2} - 3i}{\sqrt{6}}} =$$

$$P_- = |a_-|^2 = \left| \frac{3\sqrt{2} + \sqrt{3} - 3i}{\sqrt{60}} \right|^2 = \frac{1}{60} [(3\sqrt{2} + \sqrt{3})^2 + 3^2] = \frac{9 \cdot 2 + 2 \cdot 3\sqrt{6} + 9}{60}$$

$$= \frac{30 + 6\sqrt{6}}{60} = \frac{5 + \sqrt{6}}{10} \text{ каде}$$

$$P_+ = |a_+|^2 = \left| \frac{-3\sqrt{2} + \sqrt{3} + 3i}{\sqrt{60}} \right|^2 = \frac{1}{60} [(-3\sqrt{2} + \sqrt{3})^2 + 3^2] = \frac{9 \cdot 2 - 2 \cdot 3\sqrt{6} + 9}{60}$$

$$= \frac{30 - 6\sqrt{6}}{60} = \frac{5 - \sqrt{6}}{10}$$

Задача 6:

(a) Накаси иди стапорачи от \mathbb{R} за \mathbb{C} за оставените

Σταθρικ Ασένος

Σε πλανητικό καρεκτυρημ (θ, φ);
N.d.o. έχει ρινοτρύπα $\frac{1}{2}$.

Έστω το διάνομα της σφραγίδας καρεκτυρημ (θ, φ), τη μαριάσια.

$$\begin{aligned} S_n &= \frac{1}{2} \left(\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} n_x + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} n_y + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} n_z \right) = \frac{1}{2} \left(\begin{pmatrix} n_x & n_x - in_y \\ in_x + n_y & -n_z \end{pmatrix} \right) \\ S_n &= \frac{1}{2} \begin{pmatrix} \cos \theta & \sin \theta (\cos \phi - i \sin \phi) \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \cos \theta & \sin \theta e^{i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{aligned}$$

Ρινοτρύπα των στραβών $S_n = \frac{1}{2} A \mu e$

$$A = \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix}$$

Βροχολογία των ρινοτρύπων της Α:

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} \cos \theta - \lambda & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta - \lambda \end{vmatrix} = 0 \Rightarrow \cos(\cos \theta - \lambda)(-\cos \theta - \lambda) - \sin^2 \theta e^{i\phi} e^{-i\phi} = 0 \Rightarrow$$

$$\lambda^2 - \cos^2 \theta - \sin^2 \theta = 0 \Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$$

Αριθμός ρινοτρύπων $S_n: \pm \frac{1}{2} (2)$

(6) Σε διάνομα των περιπράξεων μία καράραντα με καθρούσια σφραγίδα στην ταύτιση σε αυτή την πλευρά.

$$S_n X_{+}^{(n)} \stackrel{(6)}{=} \frac{1}{2} X_{+}^{(n)} (1)$$

$$\frac{1}{2} \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} a \cos \theta + b \sin \theta e^{-i\phi} = a \\ a \sin \theta e^{i\phi} - b \cos \theta = b \end{cases} \Rightarrow$$

$$a(1 - \cos \theta) = b \sin \theta e^{-i\phi} \Rightarrow$$

$$2a \sin^2 \frac{\theta}{2} = 2b \sin \frac{\theta}{2} \cos \frac{\theta}{2} e^{-i\phi} \quad \forall \theta \Rightarrow a \sin \frac{\theta}{2} = b \cos \frac{\theta}{2} e^{-i\phi}$$

$$\text{Διαλέγεται } a = \cos \frac{\theta}{2} \text{ και } b = \sin \frac{\theta}{2} e^{i\phi} \text{ έτους σα } |a|^2 + |b|^2 = \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} = 1$$

$$e^{-i\phi} = 1$$

a

$$\text{Apa } \chi_{+}^{(n)} = \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} e^{i\phi} \end{pmatrix} \text{ para } \omega_0 \neq 0$$

$$\Im \chi_{+}^{(n)}(t) = -\frac{f}{2} \chi_{+}^{(n)}(1)$$

$$\frac{f}{2} \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -\frac{f}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} a \cos \theta + b \sin \theta e^{-i\phi} = -a \\ a \sin \theta e^{i\phi} - b \cos \theta = -b \end{cases} \Rightarrow$$

$$a(-1 - \cos \theta) = b \sin \theta e^{i\phi} \quad b(1 - \cos \theta) = -a \sin \theta e^{i\phi} \Rightarrow$$

$$2b \sin^2 \frac{\theta}{2} = -2a \sin \frac{\theta}{2} \cos \frac{\theta}{2} e^{i\phi} \quad \forall \theta \Rightarrow b \sin \frac{\theta}{2} = -a \cos \frac{\theta}{2} e^{i\phi}$$

Διαλέγουμε $a = \sin \frac{\theta}{2}$ και $b = -\cos \frac{\theta}{2} e^{i\phi}$ Εξαρτώμε τη $|a|^2 + |b|^2$

$$\geq \sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} e^{i\phi} e^{-i\phi} = 1$$

$$\text{Apa } \chi_{-}^{(n)} = \begin{pmatrix} \sin \frac{\theta}{2} \\ -\cos \frac{\theta}{2} e^{i\phi} \end{pmatrix} \text{ para } \omega_0 \neq 0$$

Άσκηση 7:

Σούχισα σε καρδιόσαν με κατερινένη σχεμή των \mathbb{R}^2 και \mathbb{R} .

$$\int_{\mathbb{R}^2} \psi^* \lambda \psi \chi \in \mathbb{R} \quad (1)$$

$$\text{Επιμελώς: } \int_{\mathbb{R}^2} \psi^* \lambda \psi \chi dx = i \int_{\mathbb{R}} \int_{\mathbb{R}} \psi^* \lambda \psi \chi dx dy = i \int_{\mathbb{R}} \left(\int_{\mathbb{R}} \psi^* \lambda \psi \chi dy \right) dx -$$

$$- \int_{\mathbb{R}} \left(\int_{\mathbb{R}} \psi^* \lambda \psi \chi dy \right) dx \stackrel{(1)}{=} i \int_{\mathbb{R}} \left(\int_{\mathbb{R}} \psi^* \lambda \psi \chi dx - \int_{\mathbb{R}} \left((\lambda \psi)^* \lambda \psi \chi \right) dx \right) \stackrel{(1)}{=}$$

$$= \frac{1}{i} \left(\lambda \int_{\mathbb{R}} \psi^* \lambda \psi \chi dx - \int_{\mathbb{R}} (\lambda \psi)^* \lambda \psi \chi dx \right) = \frac{1}{i} \left(\lambda \int_{\mathbb{R}} \psi^* \lambda \psi \chi dx - \lambda^* \int_{\mathbb{R}} \psi^* \lambda \psi \chi dx \right)$$

$$= \frac{1}{i} \left(\lambda \cancel{\int_{\mathbb{R}} \psi^* \lambda \psi dx} - \lambda^* \cancel{\int_{\mathbb{R}} \psi^* \lambda \psi dx} \right) = 0 \quad (2)$$

$$\text{Επί τόπου } \int_{\mathbb{R}^2} \lambda \psi \chi = i \int_{\mathbb{R}} \lambda \psi \chi, \lambda = \lambda^* \Rightarrow$$

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Στάρπικ Ασένος

$$\begin{aligned}
 \langle L_y \rangle &= \int_{\text{spac}} \psi^* L_y \psi d^3x = \int_{\text{spac}} \psi^* \left[\frac{1}{i\hbar} \partial_x \right] \psi d^3x = \frac{1}{i\hbar} \left(\int_{\text{spac}} \psi^* \partial_x \psi d^3x - \int_{\text{spac}} \psi^* \partial_x \psi d^3x \right) \\
 &= \frac{1}{i\hbar} \left(\int_{\text{spac}} (\lambda)^* \partial_x \psi d^3x - \lambda \int_{\text{spac}} \psi^* \partial_x \psi d^3x \right) \stackrel{(1)}{=} \frac{1}{i\hbar} \left(\lambda^* \int_{\text{spac}} \psi^* \partial_x \psi d^3x - \lambda \int_{\text{spac}} \psi^* \partial_x \psi d^3x \right) \\
 &= \frac{1}{i\hbar} \left(\lambda \int_{\text{spac}} \psi^* \partial_x \psi d^3x - \lambda \int_{\text{spac}} \psi^* \partial_x \psi d^3x \right) = 0
 \end{aligned}$$

Άρχοντας:

Ανατοληγράφων των λ γρας $|1, m\rangle$, $m = -1, 0, +1$.
Εκπρόσωποι καταστάσεων και υποψηφίων των $|1, m\rangle$,
 $m = -1, 0, +1$.

$$\begin{aligned}
 |L_x|1, m\rangle &= \frac{1+1}{2} |1, m\rangle = \frac{1}{2} [L_+|1, m\rangle + L_-|1, m\rangle] = \\
 &= \frac{1}{2} [\hbar \sqrt{1(m+1)-m(m+1)} |1, m+1\rangle + \hbar \sqrt{1(m+1)-m(m-1)} |1, m-1\rangle] = \\
 &= \frac{\hbar}{2} [\sqrt{2-m(m+1)} |1, m+1\rangle + \sqrt{2-m(m-1)} |1, m-1\rangle]
 \end{aligned}$$

Συνέπεια:

$$|L_x|1, -1\rangle = \frac{\hbar}{2} [\sqrt{2+1(1+1)} |1, -1+1\rangle + 0] = \frac{\sqrt{2}\hbar}{2} |1, 0\rangle \quad (1)$$

$$|L_x|1, 0\rangle = \frac{\hbar}{2} [\sqrt{2-0(0+1)} |1, 0+1\rangle + \sqrt{2-0(0-1)} |1, 0-1\rangle] =$$

$$= \frac{\sqrt{2}\hbar}{2} (|1, -1\rangle + |1, 1\rangle) \quad (2)$$

$$|L_x|1, 1\rangle = \frac{\hbar}{2} [\cancel{\sqrt{2-1(1+1)} 0} + \sqrt{2-1(1-1)} |1, 1-1\rangle] = \frac{\sqrt{2}\hbar}{2} |1, 0\rangle \quad (3)$$

Άρθρα (1), (2), (3) εξαρτώνται στη

$$L_x \begin{pmatrix} |1, -1\rangle \\ |1, 0\rangle \\ |1, 1\rangle \end{pmatrix} = \frac{\sqrt{2}\hbar}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |1, -1\rangle \\ |1, 0\rangle \\ |1, 1\rangle \end{pmatrix} \Rightarrow$$

$$L_x = \frac{\sqrt{2}\hbar}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{\sqrt{2}\hbar}{2} A, A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Tia va brângut căi (soluções) de la ecuație diferențială și soluțiile

$$\det(A - \lambda I) = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 1 & 0 \\ 1 & 1-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0 \Rightarrow -\lambda \begin{vmatrix} 1 & 1 \\ 1 & 1-\lambda \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 0 & 1-\lambda \end{vmatrix} + 0 = 0 \Rightarrow$$

$$\lambda((1^2 - 1)(0+1)) = 0 \Rightarrow \lambda(1 - (1^2 - 1)) = 0 \Rightarrow \lambda(1 - (2-1)) = 0 \Rightarrow \lambda(1-1) = 0 \Rightarrow \lambda(1^2 - 2) = 0 \Rightarrow (1+\sqrt{2})(1-\sqrt{2}) = 0 \Rightarrow \lambda = \sqrt{2} \text{ sau } \lambda = -\sqrt{2}$$

$$\text{Apa soluțiile de la: } -\frac{\sqrt{2}}{2}t - \frac{\pi}{4}, 0, \frac{\sqrt{2}}{2}t - \frac{\pi}{4}$$

~~Adică~~ soluții săptămânale:

$$\bullet \mu_1 = \frac{\sqrt{2}}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = -\frac{\sqrt{2}}{2} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \Rightarrow \begin{cases} \sqrt{2}a/2 = -a \\ \sqrt{2}b/2 = -b \\ \sqrt{2}c/2 = -c \end{cases} \Rightarrow a = -b = c = -\gamma$$

$$\text{Apa } \frac{\sqrt{2}}{2} \begin{pmatrix} -\gamma \\ -\gamma \\ -\gamma \end{pmatrix} = \gamma \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} = X_1$$

$$\bullet \mu_2 = \frac{\sqrt{2}}{2} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \Rightarrow \begin{cases} \sqrt{2}a/2 = a \\ \sqrt{2}b/2 = b \\ \sqrt{2}c/2 = c \end{cases} \Rightarrow a = b = c = \gamma$$

$$\because X_1 X_2 = 1 \Rightarrow \gamma \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} \gamma \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 1 \Rightarrow |\gamma|^2 (1+2+1) = 1 \Rightarrow |\gamma|^2 \cdot 4 = 1 \Rightarrow$$

$$\gamma^2 \frac{1}{4} \rightarrow$$

$$\text{Soluția generală } X = \frac{1}{2} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \text{ este soluție } -\gamma$$

$$\bullet \mu_0 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \begin{pmatrix} a \\ b \\ c \end{pmatrix} \Rightarrow \begin{cases} a=0 \\ b=0 \\ c=0 \end{cases} \Rightarrow g = a, f = b$$

$$\text{Apa } \frac{g}{2} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = a \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = X_0$$

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Πλαγικ Αριθμός

$$\langle \chi_0 | \chi_0 \rangle = 1 \Rightarrow a^* (1 \ 0 \ -1) a \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = 1 \Rightarrow |a|^2 (1+0+1)=1 \Rightarrow |a|^2 = \frac{1}{2} \Rightarrow$$

$$a = \frac{1}{\sqrt{2}} \rightarrow$$

Ιδιοκατάσταση $\chi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ με ιδιότητα 0

$\bullet M_F = +\frac{1}{2}$:

$$\frac{\sqrt{2}}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \frac{\sqrt{2}}{2} \begin{pmatrix} a \\ b \\ c \end{pmatrix} \Rightarrow \begin{cases} \sqrt{2}a/2 = a \\ \sqrt{2}a/2 + \sqrt{2}b/2 = b \\ \sqrt{2}b/2 = c \end{cases} \Rightarrow a = b, b = \sqrt{2}a/2 + \sqrt{2}a, c = \sqrt{2}a$$

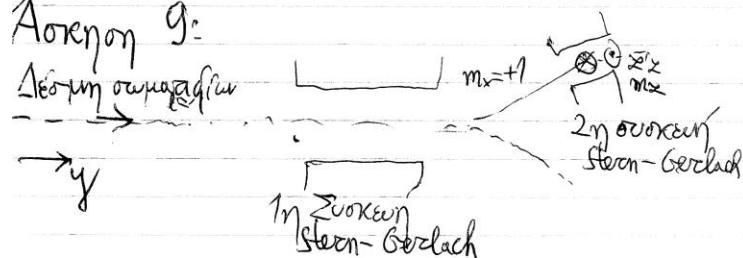
$$4pa \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a \\ \sqrt{2}a \\ a \end{pmatrix} = a \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} = \chi_F$$

$$\langle \chi_+ | \chi_+ \rangle = 1 \Rightarrow a^* (1 \ \sqrt{2} \ 1) a \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix} = 1 \Rightarrow |a|^2 (1+2+1)=1 \Rightarrow |a|^2 = \frac{1}{4} \Rightarrow$$

$$a = \frac{1}{2} \rightarrow$$

Ιδιοκατάσταση $\chi_+ = \frac{1}{2} \begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$ με ιδιότητα 1.

Αριθμός 9:



Σε πρώτη υπόθεση θα διακριτεί η δύομη μέση από την 2η συνεπαίδευση Stern-Gerlach.
Σε δεύτερης αριθμός από την κατεύθυνση

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Η δύομη ανάθεται χρησιμεύει σε δύο γραμμές υπόθεσης μία με $m_z = +1$ και μία άλλη με $m_z = -1$. Η κατεύθυνση της μέσης θα είναι στη δύομη για 50% να εγγραφεί. Από την αριθμός αριθμός αριθμός σε κατεύθυνση θα είναι ήτοι 1/2 του αριθμού αριθμού της αριθμούς