

ΜΗΧΑΝΙΚΗ ΙΙ - ΠΑΡΑΜΟΡΦΩΣΙΜΟ

ΣΗΜΕΙΩΣΕΙΣ ΔΙΑΛΕΞΕΩΝ

2012 - 2013

ΔΙΔΑΣΚΩΝ : Σ. Κ. ΚΟΥΡΚΟΥΛΗΣ
ΑΝΑΠΛ. ΚΛΗΓΝΗΤΗΣ - ΕΜΠ.

ΦΟΙΤΗΤΗΣ

[ΧΡΗΣΤΟΣ ΜΠΑΚΑΛΗΣ]
ΣΕΜΙΦΕ ΕΜΠ

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શ્રી પટેલ જી

સાધુઓની માટે કાંઈ કાંઈ

અનુભૂતિ

એ બાળ વિજય કરું હોય

Hövvedi Tıç Tıçomı

$$\sigma = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A}$$

(dilatans)

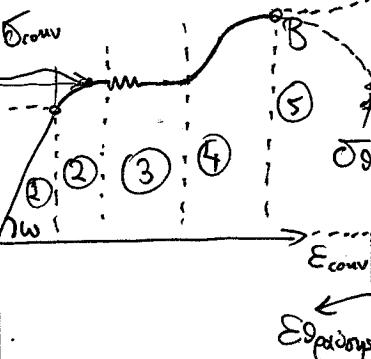
$$\epsilon = \lim_{\Delta L \rightarrow 0} \frac{\Delta L}{L} \quad [\delta = \frac{\Delta L}{L_0}]$$

① conventional

$$L \rightarrow L + \alpha \Delta \delta \epsilon_1$$

$$\sigma_{yield} = E$$

$[\delta = \epsilon E]$



1: Tedaffikî eJadrikomıya Linear Elasticity

2: Mıy - Tedaffikî Non-Linear

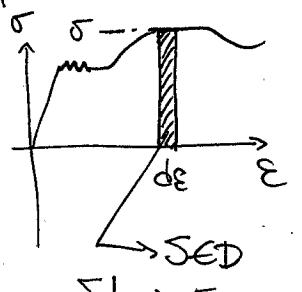
3: Nepoxiy Pois Flow Region

4: Nepoxiy Efektivipus Hardening
Kırıvıus Region

B: Ispes Dıxalıwıus
Bifurcation Point (ULTIMATE)
STRESS

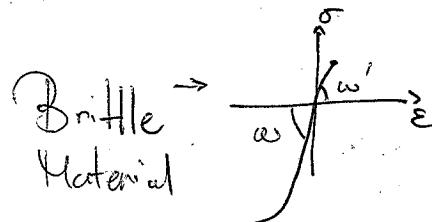
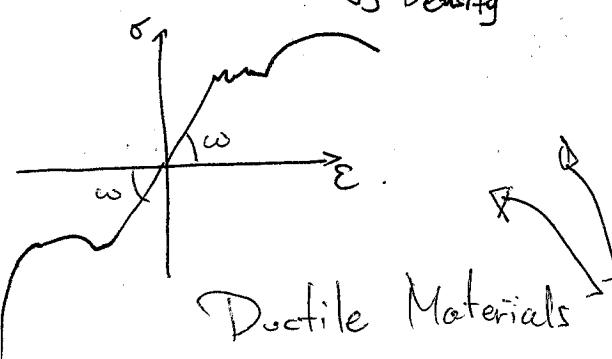
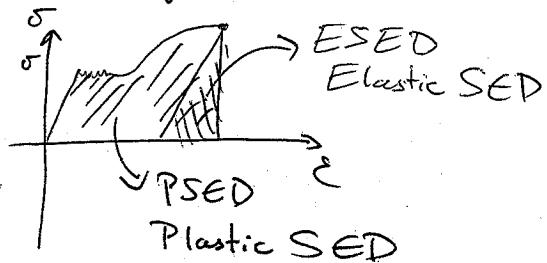
5: Nepoxiy Hafızkıvıus
Softening Region \rightarrow Elastıcılık
Kırıvıus

(Tıçm Dıxepoisi)
(Tıçm Atozoxıus)



$$\frac{\sigma - \sigma_0}{E} d\epsilon \quad \frac{F d\Delta L}{A_0} \frac{L}{L_0} = \frac{W_{epro}}{V_{epro}}$$

Strain Energy Density



Poisson

$$\epsilon = \frac{\Delta L}{L_0}$$

$$E_{tr} = \frac{\Delta D}{D_0}$$

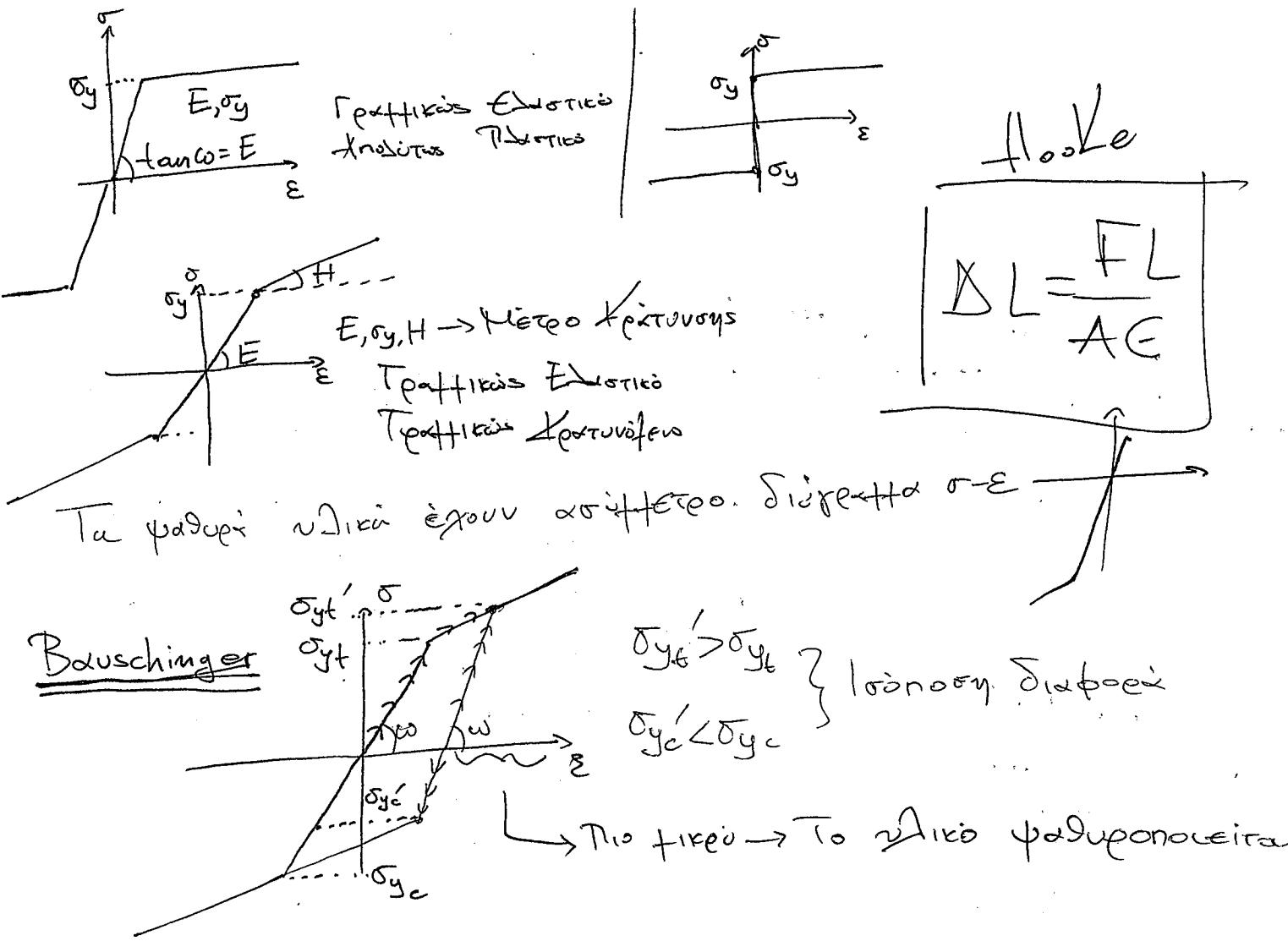
Tıçm nepoxiy Tedaffikîs eJadrikomıya
kaç MONO

$$\rightarrow \left[\nu = - \frac{\text{Etransver}}{\text{Elongitudinal}} \right]$$

$\nu \in (0, 0.5)$

(H/W) 1-6

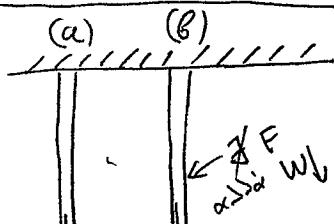
①



Ολκήτα υγρά φορητές πέραν της Τάξεως Διάρροις και αποφοριζότες δε επιβασιούν:

- (a) $\sigma_y \uparrow$ σε περιττώμα οδογύρης εναυγόρησης.
- (b) $\sigma_y \downarrow$ σε περιττώμα επεργύρης εναυγόρησης.

Στατικής Οριούσα Ανοικία Προβλήματα



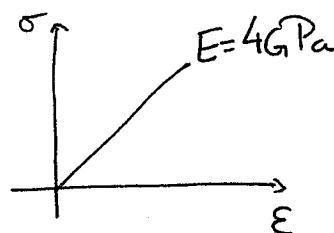
$$W=1\text{ kN}$$

$$L_0=1\text{ m}$$

$$A=10^2 \text{ mm}^2$$

(a)

$$\sigma = \frac{F}{A} = \frac{10^3}{10^2 \times 10^{-6}} = 10^7 = 10 \text{ MPa}$$



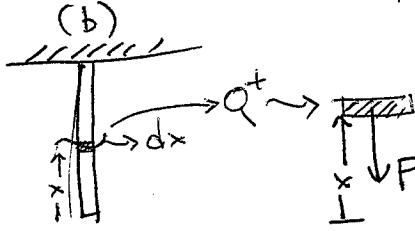
$$\epsilon = \frac{\sigma}{E} = \frac{10 \times 10^6}{4 \times 10^9} = 2.5 \times 10^{-3}$$

$$\epsilon = \frac{\Delta L}{L_0} \rightarrow \Delta L = 2.5 \times 10^3 \times 10^{-3} = 2.5 \text{ mm}$$

→

(2)

Στην περιπτώση (b) η πάροδος δεν είναι αβαρής. και φορητέα
τόσο ανά το βάρος Ιγκ.



$$\sigma(x) = \frac{F(x)}{A_0} = \frac{Wx}{LA_0}$$

$$\epsilon(x) = \frac{\Delta L}{L_0} = \frac{W}{LA_0 E} x \rightarrow \Delta L = \epsilon(x) dx = \frac{W}{LA_0 E} x dx$$

$$\Delta L = \frac{W}{LA_0 E} \int_0^L x dx$$

$$\Delta L = \frac{WL^2}{2LA_0 E} = \frac{10^3 \times 1^2}{2 \times 1 \times 10^2 \times 4 \times 10^9}$$

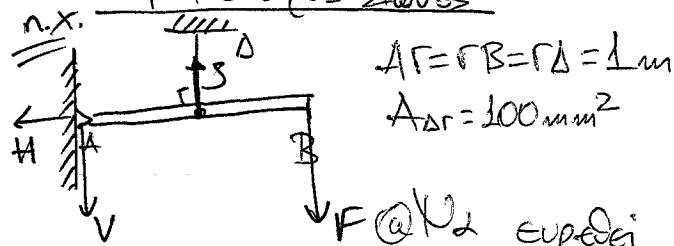
$$\boxed{\Delta L = 0.125 \times 10^{-6} \text{ m}}$$

~~ΔL = 12.5 μm~~

! Ποια είναι η τελική γεωμετρική τοποθεσία των α, β ? !

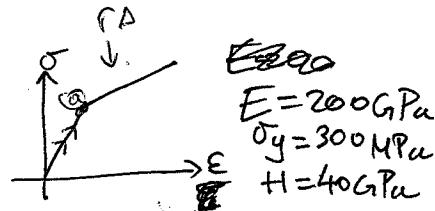
↪ Κύλινδρος

↪ Καλαμός λίνος



$$A_F = F_B = F_A = 1 \text{ kN}$$

$$A_{AB} = 100 \text{ mm}^2$$



AB → αβαρής
και σταθμός

④ Η Δευτερεύουσα εργασία είναι |F| που θα προκαλέσει αντοχιδιόνα

⑤ Η δευτερεύουσα εργασία στο δακτύο AB;

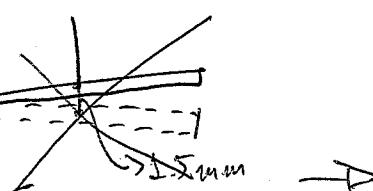
⑥ Αντίστροφα στο φορτίο χαίρει 20%. Και αποφορίζεται εργασία
η τελική δευτερεύουσα εργασία του δακτύου.

$$\textcircled{a} \sum M_A = 0 \Rightarrow 2F = S$$

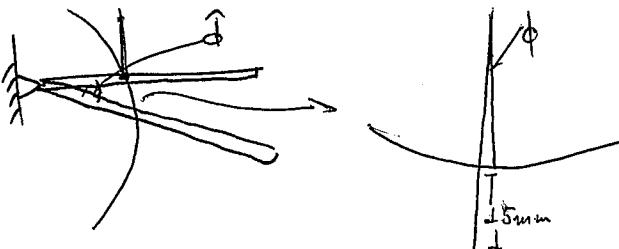
$$\sigma_{r0} \leq \sigma_y \Rightarrow \frac{S}{A_{AB}} = 3 \times 10^8 \rightarrow \frac{2F}{10^2 \times 10^{-6}} = 3 \times 10^8 \rightarrow \boxed{F_y = 15 \text{ kN}}$$

$$\textcircled{b} \frac{\sigma_y A}{E} - \epsilon \cdot E_y = \frac{300 \times 10^9}{300} \quad \epsilon_y = \frac{\sigma_y}{E} \rightarrow \epsilon_y = \frac{300 \times 10^6}{200 \times 10^9} \rightarrow \boxed{\epsilon_y = 1.5 \times 10^{-3}}$$

$$\epsilon_y = \frac{\Delta L_y}{L} \rightarrow \Delta L_y = 1.5 \times 10^3 \times 1 \rightarrow \Delta L_y = 1.5 \text{ mm}$$



(3)

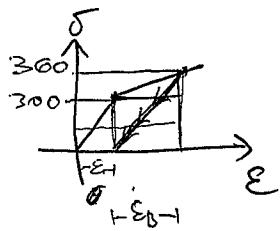


Στην πραγματικότητα το σημείο Γ δεν κατεβαίνει κατακύρωση → κατά προέγραψη
όπως την πλέουντε κατακύρωση ($\phi = 0.0866$)

$$\textcircled{1} \quad F' \rightarrow 20^\circ \uparrow \quad F' = 13 \text{ kN} \rightarrow \textcircled{2} \quad S' = 36 \text{ kN} \cdot (\text{βι. ροή})$$

$H_{\text{περιττό}}$
είναι μη.

$$\sigma' = \frac{36 \times 10^3}{10^2 \times 10^{-6}} \rightarrow \sigma' = 360 \text{ MPa}$$



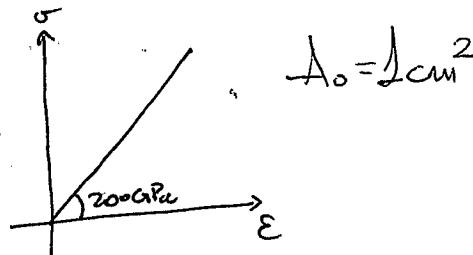
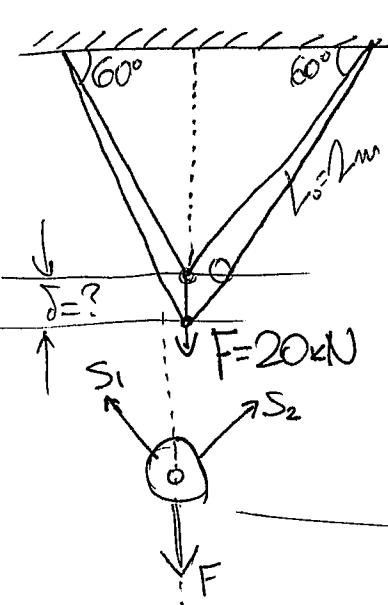
$$\textcircled{3} \quad \sigma_B = \epsilon_B H \rightarrow \epsilon_B = \frac{60 \times 10^6}{40 \times 10^3} \rightarrow \epsilon_B = 1.5 \times 10^{-3}$$

$$\epsilon_{el} = \epsilon_B + \epsilon_y = 3 \times 10^{-3}$$

$$\epsilon_{rem} = \epsilon_{yj} - \epsilon_{el} - \epsilon_{elastic}$$

$$\epsilon_{elastic} = \frac{360 \times 10^6}{200 \times 10^3} = 1.8 \times 10^{-3} \rightarrow \epsilon_{rem} = 1.8 \times 10^{-3}$$

$$\boxed{\epsilon_{rem} = 1.2 \times 10^{-3}}$$



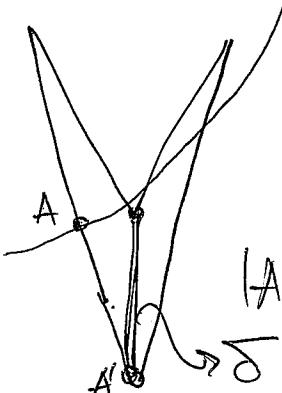
Ηδε εφεύρει η τελική δύση του κόπου Ο.

$$2S \cos 30^\circ \rightarrow S = \frac{2 \times 20}{2 \sqrt{3}} \rightarrow \boxed{S = \frac{20}{\sqrt{3}} \text{ kN}}$$

$$\sigma_s = \frac{S}{A_o} \rightarrow \sigma_s = \frac{11.56 \times 10^3}{10^{-4}} \rightarrow \sigma_s = 11.56 \times 10^7 \rightarrow \sigma_s = 115.6 \text{ MPa}$$

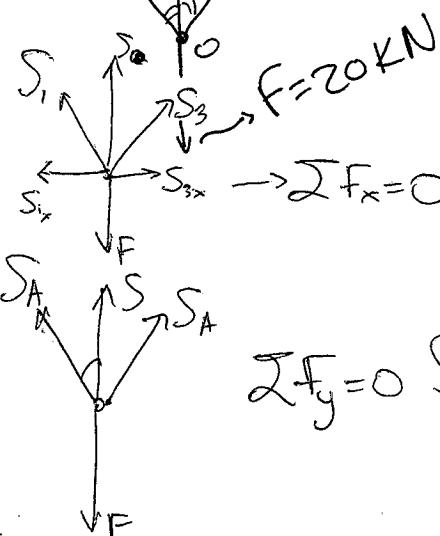
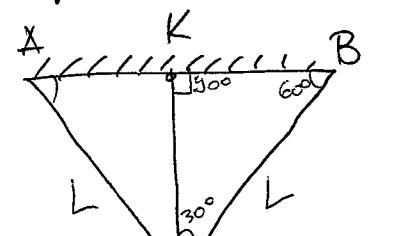
$$\epsilon_s = \frac{\sigma_s}{E} \rightarrow \epsilon_s = \frac{115.6 \times 10^6}{200 \times 10^3} \rightarrow \epsilon_s = 0.578 \times 10^{-3}$$

$$\Delta L_s = \epsilon_s L_o = 1.156 \times 10^{-3} \rightarrow \boxed{\Delta L_s = 1.16 \text{ mm}}$$



$$|AA'| = \Delta L_s \sim \cos 30^\circ = \frac{\Delta L_s}{\delta} \rightarrow \delta = \frac{1.16}{0.866} \rightarrow \boxed{\delta = 1.34 \text{ mm}}$$

| YNEPΣΤΑΤΙΚΑ ΤΠΟΒΑΗΜΑΤΑ |



$$A_0 = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$$

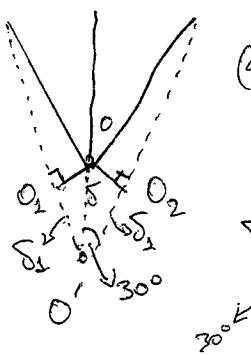
$$E = 200 \times 10^9 \text{ Pa}$$

$$L = L_{AO} = L_{OB} = 2 \text{ m}$$

$$(KO) = 2 \cos 30^\circ \rightarrow L_{KO} = \sqrt{3} \text{ m.}$$

$$\left\{ \begin{array}{l} S_{1x} = \sin 30^\circ S_1 \rightarrow S_{1x} = \frac{S_1}{2} \\ S_{3x} = \sin 30^\circ S_3 \rightarrow S_{3x} = \frac{S_3}{2} \end{array} \right\} \quad S_1 = S_3 = S_A$$

$$2F_y = 0 \quad \left\{ \begin{array}{l} F = S_A + 2S_A \cos 30^\circ \\ F = S + 2S_A \frac{\sqrt{3}}{2} \end{array} \right. \rightarrow \boxed{F = S + S_A \sqrt{3}} \quad (1)$$



$$(O_1O') = (O_2O'') = \overline{d}_1 \quad \text{συμμετρία}$$

$$(OO') = \overline{d}$$

$$\overline{d}_1 = \overline{d} \cos 30^\circ$$

$$\overline{d}_1 = \overline{d} \frac{\sqrt{3}}{2} \quad (2)$$

Yneopstatiko npotymta

Xperi-joforce fid eniai efiowry η onoia npotymta dno tmi pekatecix tou npotymta efiowry eiven η efiowry (2).

$$\Delta COB: \overline{d}_1 = \frac{S_A}{A_0} \rightarrow S_A = \overline{d}_1 A_0 \xrightarrow{\text{Hooke}} S_A = \overline{d}_1 E A_0$$

$$\overline{d}_1 = \frac{\overline{d}}{L} \rightarrow \overline{d}_1 = \frac{\overline{d}}{\frac{2}{\sqrt{3}}} \rightarrow \overline{d}_1 = \frac{\overline{d} \sqrt{3}}{4}$$

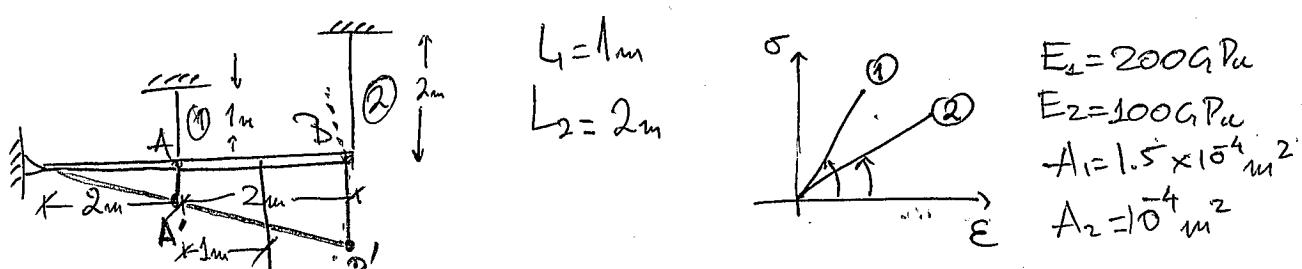
$$\overline{d}_1 = 0.44 \overline{d}$$

$$\text{KO:} \quad \rightarrow \quad \rightarrow S = \overline{d} E A_0$$

$$\overline{d} = \frac{\overline{d}}{L_{KO}} \rightarrow \overline{d} = \frac{\overline{d}}{\sqrt{3}} \rightarrow \boxed{\overline{d} = 0.58 \overline{d}}$$

$$\Rightarrow \left. \begin{array}{l} S_A = E A_0 \overline{d} 0.44 \\ S = E A_0 \overline{d} 0.58 \end{array} \right\} \xrightarrow{(1)} \overline{d} = \overline{d} E A_0 \quad f = E A_0 \underbrace{\left(0.58 \overline{d} + 0.44 \times \sqrt{3} \right) \overline{d}}_{1.34 \overline{d}}$$

$$\overline{d} = \frac{F}{E A_0} \cdot \frac{1}{1.34} \rightarrow \overline{d} = \frac{20 \times 10^3}{200 \times 10^9 \times 10^{-4}} \cdot \frac{1}{1.34} \rightarrow \boxed{\overline{d} = 0.75 \text{ mm}}$$



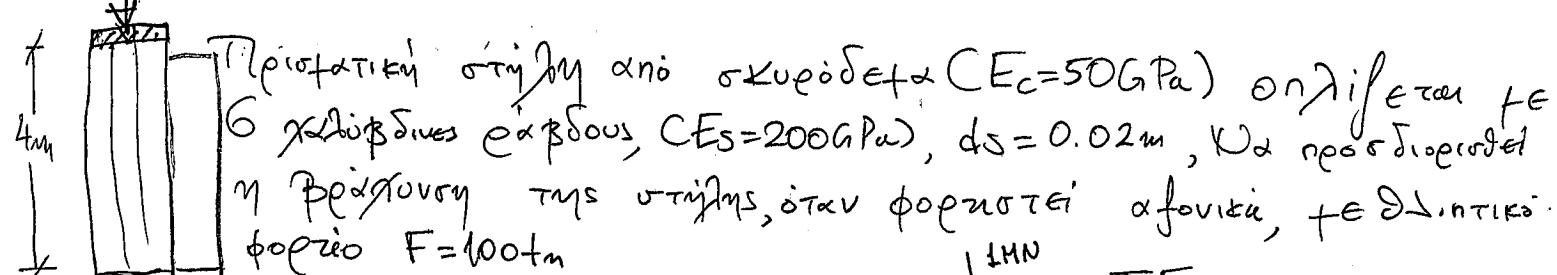
$$\begin{aligned} \sum F_x &= 0 \Rightarrow H = 0 \\ \sum F_y &= 0 \Rightarrow V + F_1 + F_2 = 30 \\ \sum M_o &= 0 \rightarrow 2F_1 + 4F_2 = 90 \quad (1) \end{aligned}$$

$$[OA \approx OBB' \quad \frac{AA'}{BB'} = \frac{1}{2}] \Rightarrow \frac{\Delta L_1}{L_1} = \frac{1}{2} \quad \frac{\Delta L_1}{\Delta L_2} = \frac{1}{2} \quad \frac{\Delta L_1}{\Delta L_2} = \frac{1}{2} \Rightarrow \frac{2F_2}{\frac{1 \times 100}{1.5 \times 200}} = 2 \Rightarrow F_1 = 3F_2 \quad (2)$$

(1) + (2) \Rightarrow Βρεισκω συνέπεια τοξεις \rightarrow σποντ.

$$6F_2 + 4F_2 = 90 \Rightarrow F_2 = 9 \text{ kN} \rightarrow F_1 = 27 \text{ kN} \rightarrow \sigma_1 = \frac{F_1}{A_1} = \frac{27 \times 10^3}{1.5 \times 10^{-4}} \rightarrow \sigma_1 = 180 \text{ MPa}$$

$$\sigma_2 = \frac{F_2}{A_2} = \frac{9 \times 10^3}{10^{-4}} \rightarrow \sigma_2 = 90 \text{ MPa}$$

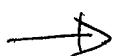


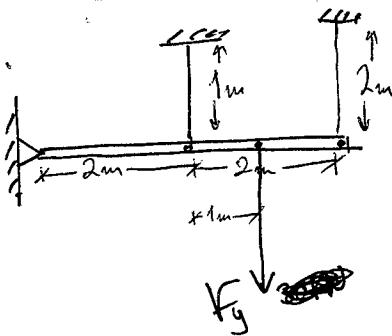
$$\begin{aligned} 1 \text{ tN} &\rightarrow 1000 \text{ kP} \\ 1 \text{ tN} &\rightarrow 10 \text{ kN} \rightarrow 100 \text{ tN} = F = 10^3 \text{ kN} \\ F &= 1 \text{ MN} \end{aligned}$$

$$\begin{aligned} \sum F_y &= 0 \\ F_c + F_s &= 1 \text{ MN} \quad (1) \end{aligned}$$

$$\left[\Delta L_c = \Delta L_s \right] \Rightarrow \frac{F_c K}{A_c E_c} = \frac{F_s K}{A_s E_s} \Rightarrow \frac{F_c}{A_c} \frac{6 \text{ nds}^2}{2 \times 10^{-11}} = \frac{F_s}{A_s} \left(0.5 \times 0.7 - \frac{6 \text{ nds}^2}{4} \right) \times 50 \times 10^9$$

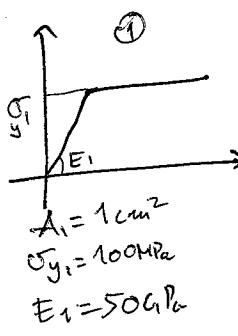
ds γνωριο \Rightarrow Βρεισκω συνέπεια.





$$L_1 = 1\text{m}$$

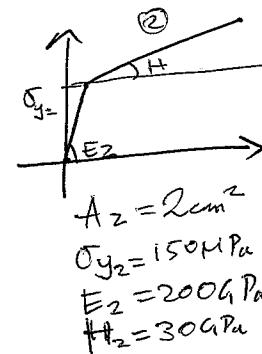
$$L_2 = 2\text{m}$$



$$A_1 = 1 \text{ cm}^2$$

$$\sigma_{y1} = 100 \text{ MPa}$$

$$E_1 = 50 \text{ GPa}$$



$$A_2 = 2 \text{ cm}^2$$

$$\sigma_{y2} = 150 \text{ MPa}$$

$$E_2 = 200 \text{ GPa}$$

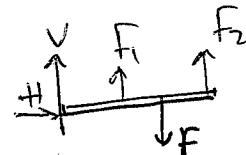
$$H_2 = 30 \text{ GPa}$$

① Η αρχεί η φόρηση F_y να ονοια θα προκαλέσει την πρώτη κατάργηση της πρώτης εκ των στυρίκτικων ράβδων και να βρεθεί η δέσμη της δοκού που στήριξε την.

② Η ΔL_2 ευρεθεί το φορτίο που θα προκαλέσει κατάργηση της δεύτερης ράβδου. Ποια η δέσμη της δοκού;

③ Η ΔL_2 λαμβάνει η διατύπωση του προηγουμένου ερωτήσεως κατά 20%. Και στη συνέχεια το συντηφθεί με τη συντηφθεί με τη συνέχεια.

Είναι;?



$$\sum F_y = 0 \rightarrow V + F_1 + F_2 = F_y$$

$$\sum M_0 = 0 \Rightarrow (F_1 + 4F_2 = 3F_y) / (1)$$

$$\cancel{2F_1 + 4F_2 = 3F_y} \quad \cancel{2 \cdot 5 \cdot A_1 + 4 \cdot 5 \cdot A_2 = 3F_y} \Rightarrow \cancel{C_1 A_1 + 2 C_2 A_2 = 3F_y}$$

$$\frac{\Delta L'}{BB'} = \frac{1}{2} \rightarrow \frac{\Delta L_1}{BL_2} = \frac{1}{2} \rightarrow \Delta L_2 = 2 \Delta L_1 \xrightarrow[100 \text{ GPa}]{\text{σύγκριση, καθώς είναι το ορικό σύγκριση, οπού τις είναι την δύο ίδια κατάργηση}} \frac{2F_2}{2 \times 200} = \frac{2F_1}{50} \rightarrow 5F_2 = 40F_1 \rightarrow \boxed{F_2 = 8F_1}$$

$$(1) \quad 2F_1 + 32F_1 = 3F_y \rightarrow \boxed{F_1 = 0.088F_y} \rightarrow \boxed{F_2 = 0.706F_y}$$

$$\rightarrow \sigma_1 = \frac{0.088}{10^4} F_y \rightarrow \boxed{\sigma_1 = 0.88F_y \text{ KN}} \quad ? \quad \rightarrow \boxed{\frac{\sigma_2}{\sigma_1} = 40} \quad \rightarrow \boxed{\frac{\sigma_2}{\sigma_1} = 3}$$

Πρώτη κατάργηση η ράβδος 2.

Δέχεται 4η δοσή φόρηση την ο διστούς των τανεών κατάργησης είναι 3.

$$\left[\begin{array}{l} \frac{\sigma_2}{\sigma_1} = A \\ \frac{\sigma_{2y}}{\sigma_{1y}} = B \end{array} \right]$$

$$A > B \quad 2 \text{ πρώτη}$$

$$A < B \quad 1 \text{ πρώτη}$$

$$\sigma_2 = 150 \times 10^3 \text{ kPa}$$

$$3.53 F_y = 150 \times 10^3 \Rightarrow F_y = 42.5 \text{ kN}$$

$$\text{Dekry: } \Delta L_2 = \frac{F_2 L_2}{A_2 E_2} = \frac{150 \times 10^6 \cdot 2}{2 \times 10^{11}} \rightarrow \Delta L_2 = 1.5 \text{ mm}$$

$$\textcircled{2} \quad \Delta L_2 = 2 \Delta L_1$$

$$\varepsilon_2 L_2 = 2 \frac{F_1' L_1}{A_1 E_1}$$

Pároforos 2
Láterixies o floute

↳ Briseisoufe to ε_2 tisw tou fítpou kptuvouys.

$$\frac{\sigma_{y1}}{E} \rightarrow \left(\frac{150}{200} \times 10^3 + \left(\frac{F_2'}{A_2} - 150 \times 10^6 \right) \times \frac{1}{30 \times 10^9} \right) L_2 = 2 \frac{F_1' L_1}{A_1 E_1}$$

$$\varepsilon_{\sigma_y} \quad \varepsilon_2 \quad \varepsilon_u$$

$$\Rightarrow \left(0.75 \times 10^{-3} + \left(2F_2' \times 10^{-3} - 150 \times 10^6 \right) \times \frac{1}{30 \times 10^9} \right) \times 2 = \frac{2F_1'}{10^4 \times 5 \times 10^{10}} \Rightarrow$$

~~$$(0.75 \times 10^{-3} + 2F_2' \times 10^{-3}) \times 2 = \frac{2F_1'}{10^4 \times 5 \times 10^{10}}$$~~

~~$$0.75 \times 10^{-3} = 0.75 \times 10^{-3} + 10^7 F_2' \times 10^{-3} \Rightarrow F_2' = 0$$~~

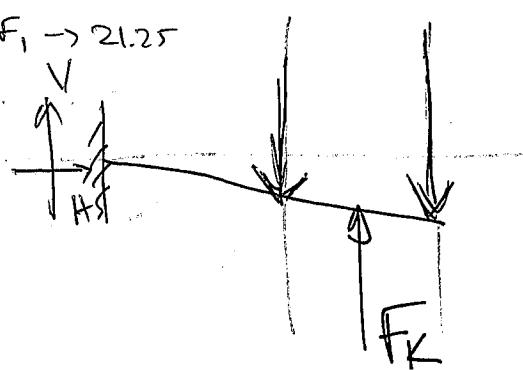
$$\Rightarrow \left(0.75 \times 10^{-3} + \frac{1}{15} \times 10^{-6} - 5 \times 10^{-3} \right) \times 2 = \frac{2F_1'}{10^4 \times 5 \times 10^{10}} \quad (1), \quad F_1' =$$

$$1.5 \times 10^{-3} + \frac{2}{15} \times 10^{-6} - 10 \times 10^{-3} = \frac{2F_1'}{5 \times 10^6}$$

$$-8.5 \times 10^{-3} + 7.5 \times 10^{-6} = 0.4 F_1' \times 10^6$$

$$-8.5 \times 10^{-3} + 7.5 = 0.4 F_1' \rightarrow 21.25$$

! Πώς ανοφεριμό;



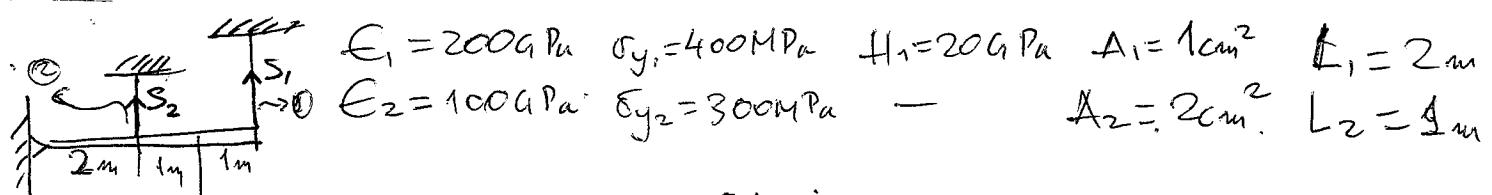
Dev αφορώ to F_k .

Aσκή διατίκο φορτίο,
 $F'_k = -F_k$.

(Μετατρέπεται σε γραμμικό φορτίο
πάρθε δε, είναι ως διάγη!)

Αλώ βότο

③



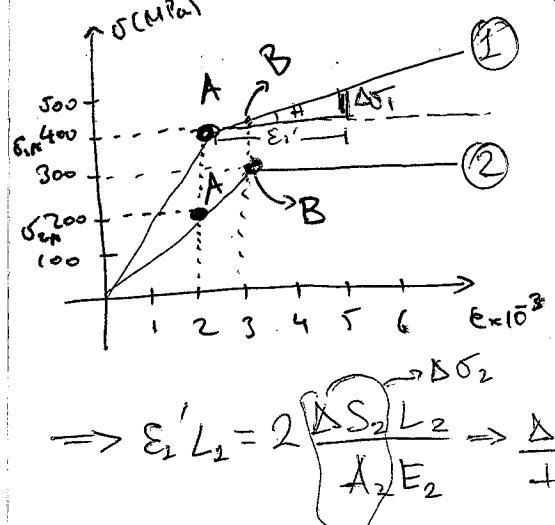
$$2S_2 + 4S_1 = 3F^{(1)} \quad [\Delta L_1 = 2\Delta L_2] \rightarrow \frac{S_1 L_1}{A_1 E_1} = 2 \frac{S_2 L_2}{A_2 E_2}$$

R
ΑΥΤΗ ΚΩΣΤΙΣΤΗΚΕ ΣΤΟΝ ΦΕΡΝΕΣ.

$$(1) \rightarrow 2S_2 + 4S_1 = 3F \rightarrow 2S_2 = F \rightarrow \sigma_2 = \frac{F}{L_2} 10^4 \text{ N} \quad \sigma_1 = \frac{F}{L_1} 10^4 \text{ N}$$

$$\frac{\sigma_1}{\sigma_2} > \frac{\sigma_{1y}}{\sigma_{2y}} \rightarrow \text{Άριστος κείμενος} \quad (1) \quad \text{η πάση.}$$

$$\sigma_{1y} = \sigma_1 \Rightarrow F = 80 \text{ kN} \quad \left[\sigma_1 = 400 \text{ MPa} \right] \quad \text{και} \quad \left[\sigma_2 = 200 \text{ MPa} \right]$$



(B) Αυξιεύοντας δύναμη ΔF, έτσι ώστε να αριστώσει με την ίδια παρόποδος (2).

$$2\Delta S_2 + 4\Delta S_1 = 3\Delta F \quad (3)$$

$$[(\Delta L_1)' = 2(\Delta L_2)']$$

$$\Rightarrow \varepsilon_1' L_1 = 2 \frac{\Delta S_2 L_2}{A_2 E_2} \Rightarrow \frac{\Delta \sigma_1}{H_1} L_1 = 2 \frac{\Delta S_2}{A_2} \frac{L_2}{E_2} \Rightarrow \frac{\Delta \sigma_1}{A_1 H_1} L_1 = 2 \frac{\Delta S_2}{A_2} \frac{L_2}{E_2}$$

$$2 \frac{\Delta S_1}{1 \times 20} = 2 \frac{\Delta S_2}{2 \times 100} \rightarrow \Delta S_1 = 0.1 \Delta S_2 \quad (3) \rightarrow 2,4 \Delta S_2 = 3 \Delta F \rightarrow \Delta S_2 = 1.25 \Delta F$$

$\hookrightarrow \Delta S_2 = 60 \text{ kN}$

$$\frac{\Delta S_2}{A_2} = \frac{1.25 \Delta F}{2 \times 10^{-4}} \Rightarrow \Delta \sigma_2 = 6250 \Delta F \quad \frac{\Delta \sigma_2 = 100 \text{ MPa}}{\Delta F = 16 \text{ kN}}$$

$$\Delta S_1 = 2 \text{ kN}$$

$$\sigma_1' = 42 \text{ kN}$$

$$F_T = F + \Delta F \rightarrow F_T = 96 \text{ kN.}$$

$$\begin{aligned} \cancel{\Delta S_2 = 60 \text{ kN}} \rightarrow \cancel{\Delta \sigma_2 = 62 \text{ kN}} \\ \uparrow \sigma_2 = 60 \text{ kN} \quad \uparrow \sigma_1' = 42 \text{ kN} \\ \cancel{\Delta S_2 = 60 \text{ kN}} \rightarrow \cancel{\Delta \sigma_2 = 62 \text{ kN}} \end{aligned}$$

$$\rightarrow \sigma_2' = 300 \text{ MPa}$$

$$\left[\varepsilon_2' = 3 \times 10^{-3} \right]$$

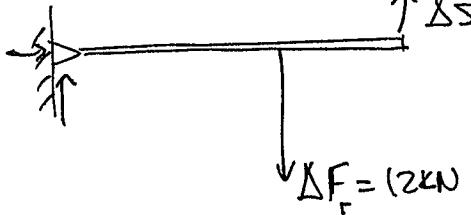
$$\sigma_1' = \frac{42 \times 10^3}{10^4} \rightarrow \sigma_1' = 420 \text{ MPa}$$

$$\varepsilon_1' = 2 \times 10^{-3} + \frac{420 - 400}{20 \times 10^3} \times 10^6 \rightarrow \left[\varepsilon_1' = 3 \times 10^{-3} \right]$$

$\sum \varepsilon_i'$
 $\Sigma MTS \Sigma EH$

$$\begin{aligned} \varepsilon_2' \rightarrow \Delta L_2' = 1 \text{ mm} \leftarrow \Delta L_2 = 3 \text{ mm} \\ \varepsilon_1' \rightarrow \Delta L_1' = \Delta L_2' = 6 \text{ mm} \end{aligned}$$

① Aufbau zu Sicherheitskraft 12 kN. C12+



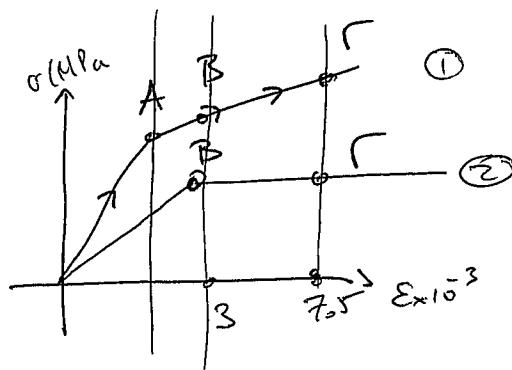
Πλέον $\exists \Delta S_2$. Είναι χρειασμένη μετατροπή.
(ΤΡΟΣΟΧΗ $\rightarrow \exists \Sigma_2 = 60 \text{ kN}$ Ανά δευτεροβάθμιο)

$$4(\Delta S'_1) = 12 \cdot 3 \rightarrow \Delta S'_1 = 9 \text{ kN}$$

$$\textcircled{1} \rightarrow F_3 = 108 \text{ kN}$$

$$S'_1 = 51 \text{ kN} (S_1 + \Delta S_1)$$

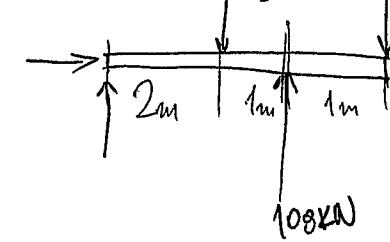
$$S'_2 = 60 \text{ kN} = S^B_2$$



$$\sigma_{\text{add}} = \frac{S'_1}{A_1} = 510 \text{ MPa} \sim \boxed{\varepsilon_{\text{add}} = 4.5 \times 10^{-3} = \varepsilon_{2 \text{ add}}}$$

ΟΣΟ ΠΑΡΑΜΟΡΦΩΝΕΤΑΙ Η ΜΙΑ, ΆΛΛΟ ΤΟΣΟ Η ΆΛΛΗ
Ενέργεια "fektiv" άνταξεις
ιδίως υψηλές τους σιδηρόπλαστα
 $\Rightarrow \varepsilon_{T_1} = \varepsilon_{T_2} = 7.5 \times 10^{-3}$

② Αποφόρηση. $\Rightarrow D_1 \vee D_2$ στην αντίτυπη διαστημή $F = 108 \text{ kN}$

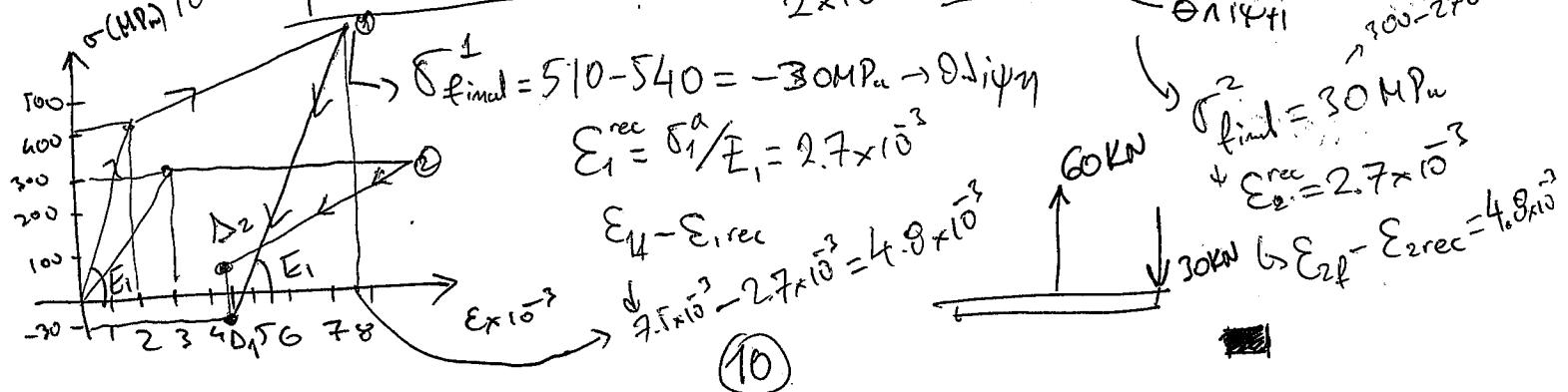


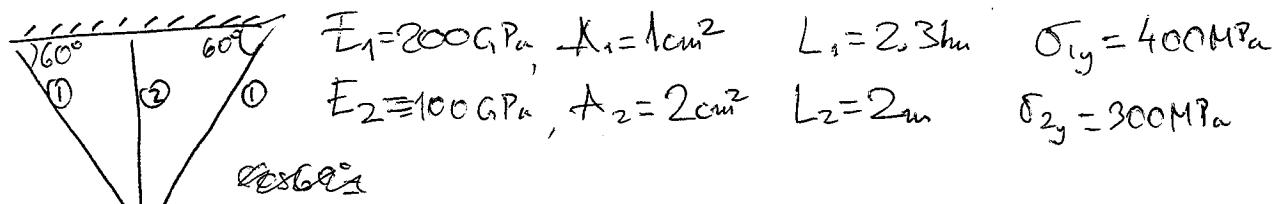
↳ Ενέργεια η αποφόρηση είναι ελαστική,
τοχεύει στην θέση της. Θα αποφευγούν
παραδόγες τε το E.

$$2\Delta S_2^a + 4S_1^a = 324 \quad (1)$$

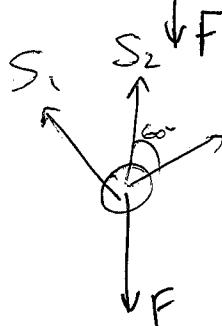
$$\Delta L_1 = 2\Delta L_2 \Rightarrow \frac{S_1^a L_1}{A_1 E_1} = \frac{S_2^a L_2}{A_2 E_2} \Rightarrow S_1^a = S_2^a \quad \left. \begin{array}{l} 6S_2^a = 324 \\ S_2^a = 54 \text{ kN} = S_1^a \end{array} \right\} \text{ΘΛΙΠΤΗΚΕΣ}$$

$$\sigma_1^a = \frac{54 \times 10^3}{10^4} \xrightarrow{\text{ΘΛΙΨΗ}} \sigma_1^a = -540 \text{ MPa} \quad \sigma_2^a = \frac{54 \times 10^3}{2 \times 10^4} \xrightarrow{\text{ΘΛΙΨΗ}} \sigma_2^a = -270 \text{ MPa}$$





Die Joule und des Toximeli für die zwei Papiere.



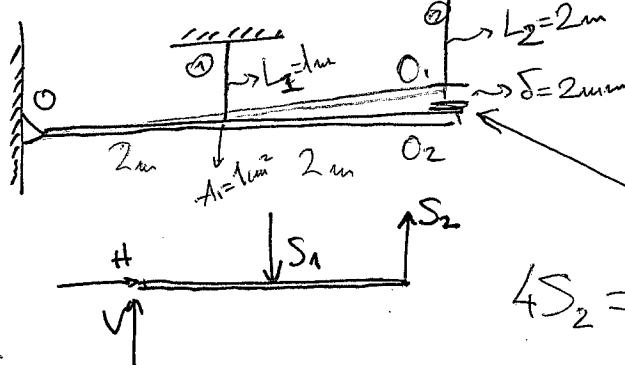
$$F = S_2 + 2S_1 \cos \overset{\curvearrowleft}{60^\circ} \underset{130^\circ}{\rightarrow} \overline{F - S_2 + S_1}$$

$$F = S_2 + \sqrt{3} S_1$$

$$\cos 30^\circ = \frac{\Delta L_1}{\Delta L_2}$$

$$\frac{dV/I}{dV} = \frac{\sqrt{3}}{2.31} S_1 S_2$$

ΠΡΟΒΛΗΜΑΤΑ ΑΕΩΝΙΚΑ ΚΕ ΚΑΤΑΣΚΕΥΑΣΤΙΚΑ ΣΦΛΗΜΑΤΑ



Οὐδὲ εὔπειρον οὐδὲ οἱ τίσεις οὐδὲ πίστεις
ναι τὴν τηγανικὴν κατανόησαν οἵτις
Σύντομος.

$$\text{Hausdorffsche Definition} \Rightarrow O_1 \cap O_2 = \emptyset$$

$$4S_2 = 2S_1 \rightarrow \boxed{S_1 = 2S_2} \quad (2)$$

Die exakte Kat. gefügt zu Supporto zu Pickout erkennbar
die Kandidaten zu λ =>

$$\begin{array}{c} O \\ O \\ \vdots \\ O_2 \end{array} \quad \delta = 2 \times (\bar{O}^3) = (OO_1) + (OO_2) \rightarrow 2 \times (\bar{O})^3 = \frac{S_2 L_2}{A_2 E_2} + 2 \frac{S_1 L_1}{A_1 E_1}$$

$\hookrightarrow \Delta L_2$

$2 \Delta L_1 \rightarrow$ ηασο κάντυνε το ①

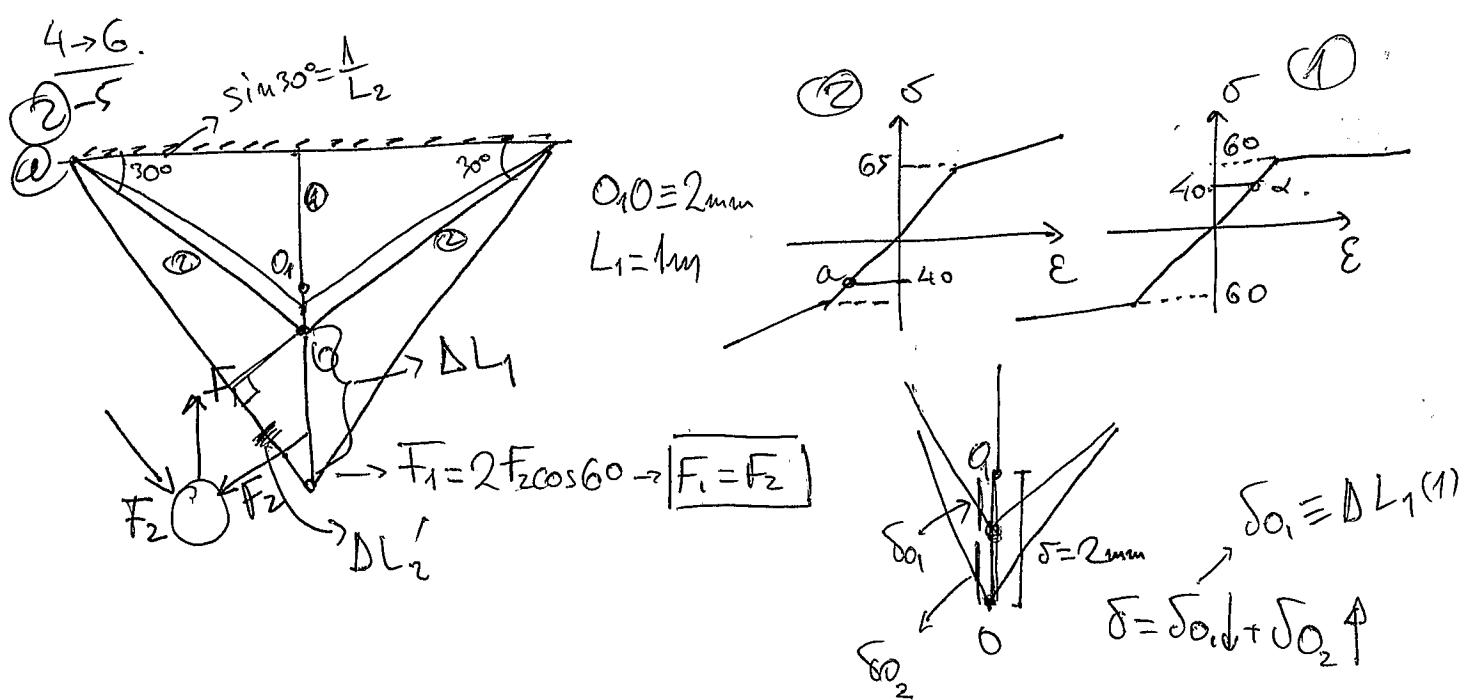
$\hookrightarrow A\&O\ OMOIA\ TPI\ O\ N\ A$

$$\rightarrow 2 \times \bar{O}^3 = \frac{2S_2}{10 \times 10^{11}} + \frac{2S_1}{2 \times 10 \times 10^{11}} \rightarrow 2 \times O^4 = S_1 + S_2 \quad (1)$$

$$\begin{array}{c} 1 \\ + \\ 2 \end{array} \left| \begin{array}{l} S_2 = 6.6 \text{ kN} \\ S_1 = 13.3 \text{ kN} \end{array} \right|$$

$$S_1 = 13.3 \text{ kN} \quad \rightarrow \sigma_1 = \frac{13.3 \times 10^3}{1 \times 10^{-4}} \rightarrow \boxed{\sigma_1 = 133 \text{ MPa}} \quad | \text{ σ Διψη}$$

$$S_2 = 6.6 \text{ kN} \quad \rightarrow \sigma_2 = \frac{6.6 \times 10^3}{1 \times 10^{-4}} \rightarrow \boxed{\sigma_2 = 66 \text{ MPa}} \quad | \text{ εφεκυσθος}$$



$$\frac{(2)}{(1)} \rightarrow \delta = \frac{F_1 L_1}{A_1 E_1} + \frac{F_2 L_2}{\cos 60^\circ A_2 E_2} \rightarrow 2 \times 10^3 = \frac{F_1 \cdot 1}{(10 \times 10^9 \times 10^{-4})} + \frac{F_2 \cdot 1}{\cos 60^\circ \times (10 \times 10^9 \times 10^{-4})}$$

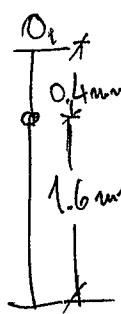
$$\cos 60^\circ = \frac{\Delta L_2}{\delta_{O_2}} \quad (2)$$

$$\sigma_1 = \sigma_2 = \frac{4 \times 10^3}{10^{-4}} = 40 \text{ MPa}$$

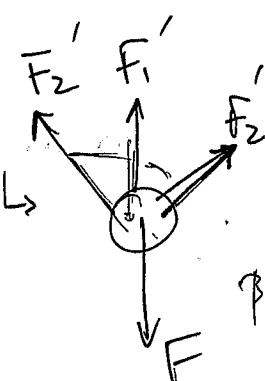
$$\therefore \boxed{F_1 = F_2 = 4 \text{ kN}}$$

Οι πίεσοι 2 έχουν τραχυνθεί. Η 1 έχει αντιγραφεί.

$$\textcircled{B} \quad \Delta L_1 = \frac{\sigma L_1}{E} = \frac{40 \times 10^6}{100 \times 10^9} = 0.4 \text{ mm}$$



Δεν ισχει $F_1 = F_2$.



$$F_1' + 2F_2' \cos 60^\circ = F \rightarrow \boxed{F_1' + F_2' = F}$$

$$\textcircled{B}, \textcircled{C}, \textcircled{D}, \textcircled{E} \rightarrow \Delta L_2 = \Delta L_1 \frac{1}{2} \rightarrow \boxed{2 \Delta L_2 = \Delta L_1}$$

L v. Hooke

(12)

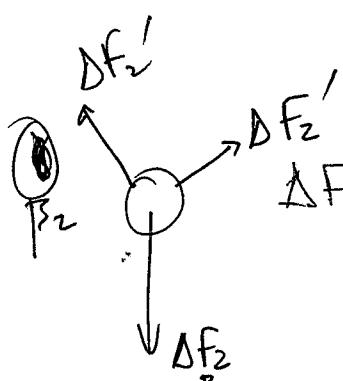
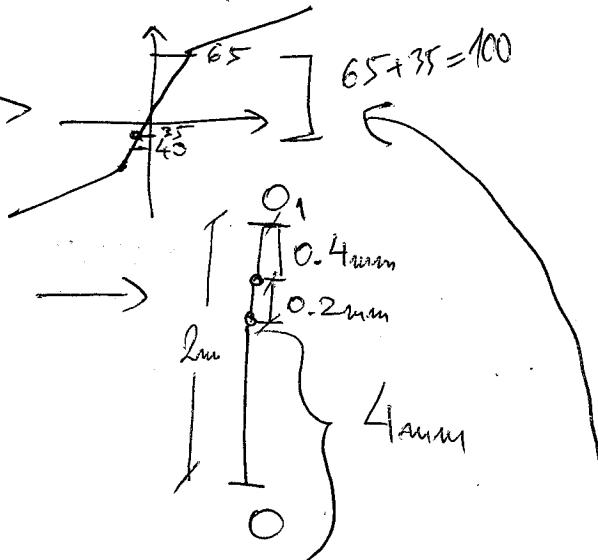
$$F_1' = 4F_2 \rightarrow \sigma_1' = 4\sigma_2$$

$$F_1' + F_2' = F \quad !\sigma_{y1}' = 20 \text{ MPa}!$$

$$! \sigma_{y2}' = 105 \text{ MPa}! \quad \left. \begin{array}{l} \text{In der Regel ist } \\ \text{die Spannung } \sigma_1' \text{ gegeben.} \end{array} \right\} \sigma_1' = 20 \times 10^6 \rightarrow F = 2.5 \text{ kN.}$$

$$\sigma_2 = \frac{20 \times 10^6}{4} \rightarrow \sigma_2 = 5 \text{ MPa}$$

$$\Delta L_1' = \frac{20 \times 10^6}{100 \times 10^9} \sim \Delta L_1' = 0.2 \text{ mm.}$$



$$\Delta F_{\beta_2}' = 2\Delta F_2' \cos 60^\circ \rightarrow \frac{\Delta F_{\beta_2}}{A} = \frac{\Delta F_2'}{A} \rightarrow \Delta \varepsilon = \frac{\Delta F_{\beta_2}}{A} = 100 \times 10^{-6}$$

$$\cos 60^\circ = \frac{\Delta L_2'}{\Delta L_1'} \rightarrow \Delta L_2' = \Delta \varepsilon E_2 L_2 = \frac{10^8}{100 \times 10^9} \times 2 \rightarrow \Delta L_2' = 2 \text{ mm}$$

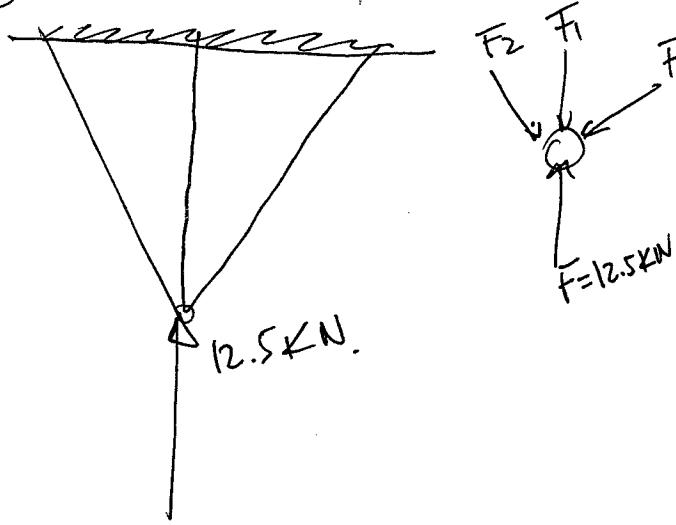
$$\boxed{\Delta F_{\beta_2} = 10 \text{ kN}}$$

$$\Delta L_1' = 4 \text{ mm}$$

$$\Delta \varepsilon_1 = 4 \times 10^{-6}$$

$$\Delta \varepsilon_{\alpha_1} = \varepsilon_1 + \varepsilon_y = 4.6 \times 10^{-6}$$

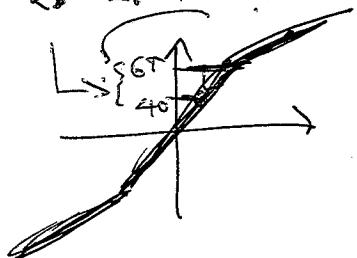
(B) - Aoki + fix iron can durability SinterfM -



$$12.5 \times 10^3 = F_1^a + F_2^a$$

$$2\Delta L_2^a = \Delta L_1^a$$

$$\left. \begin{array}{l} F_1^a = 10 \text{ kN} \rightarrow 100 \text{ MPa} \\ F_2^a = 2.5 \text{ kN} \rightarrow 25 \text{ MPa} \end{array} \right\}$$



(4-5)

$$\sum F_y = 0 \rightarrow S_1 \sin 60^\circ + S_2 = F$$

$$S_1 = \frac{F}{\sin 60^\circ} = \frac{F}{\sqrt{3}}$$

$$\Delta L_{Eo} = \frac{\Delta L_{AO}}{\sin 60^\circ} = \frac{\Delta L_{AO}}{\sqrt{3}}$$

(2-3)

$$\int \sigma_y dA = f(x) A dx$$

$$\int \sigma_y \frac{dA}{A} = \int f(x) dx$$

$$\ln \frac{A(x)}{A_0} = f(x) x \Rightarrow \ln \frac{r(x)}{r_0} = f(x) x \Rightarrow r(x) = r_0 e^{\int f(x) x}$$

ΘΕΡΜΟΚΡΑΣΙΑΚΕΣ ΜΕΤΑΒΟΛΕΣ

$$\Delta L = \alpha L_0 (\Delta T)$$

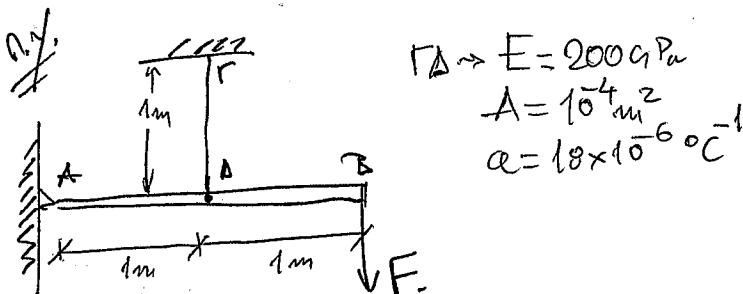
$$\alpha = \frac{\Delta L}{L_0 \Delta T} \text{ [} ^\circ \text{C} \text{ }]^{-1}$$

$$\epsilon = \frac{\Delta L}{L_0} \quad \sigma = \epsilon E = \frac{\Delta L}{L_0} E = \frac{\alpha L_0}{L_0} (\Delta T) E$$

$$\sigma = \alpha (\Delta T) E$$

$$\boxed{\sigma = \alpha (\Delta T) E}$$

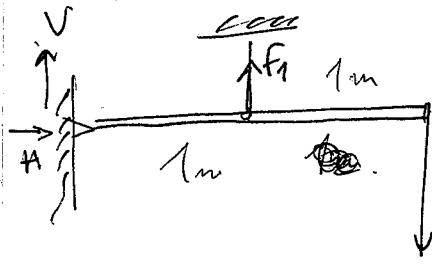
$$\boxed{\Delta L = \alpha L_0 (\Delta T)}$$



Έσω στη συγκεκρινή θέση της Β πάρτε την δύναμη F. Φixate τη γραμμή ΑΒ.

~ Να ευπεδεί μη σχειρό τεταφού F και ΔΤ ωρίτε μη η γραμμή ΑΒ να μεριμνεί οριζόντια.

14



$$F_1 = 2F$$

$$\sigma_g = \alpha E(\Delta T)$$

$$F_1 = \alpha A_1$$

$$\alpha E(\Delta T) A_1 = 2F$$

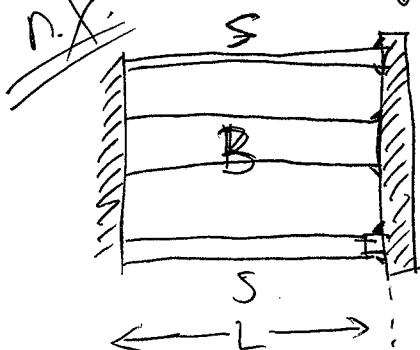
$$10 \times 10^{-6} \times 200 \times 10^9 \times 10^{-4} (\Delta T) = 2F$$

$$2F = 360 (\Delta T) \rightarrow F$$

~~F = 180 (\Delta T)~~

$$\boxed{F(\Delta T) = 180 \Delta T}$$

Kanizki



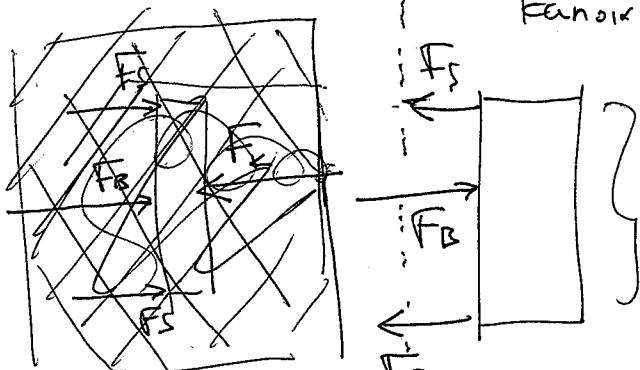
$$S: E_S = 200 \text{ GPa}, \alpha_S = 12 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}, \sigma_y S = 200 \text{ MPa}$$

$$B: E_B = 100 \text{ GPa}, \alpha_B = 18 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}, \sigma_y B = 80 \text{ MPa}$$

$$A_B = 2A_S$$

~ Auf einer Temperatur T variiert ΔT . $\rightarrow \sigma_S = f(\Delta T)$?
 $\rightarrow \sigma_B = f(\Delta T)$?

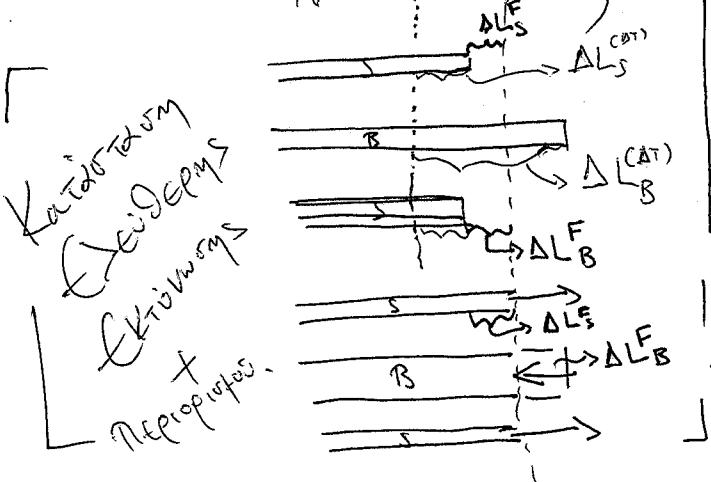
~ Noch unklar ob es ein z. B. $\Delta T = T_{\text{ref}} - T$ und α_{ref} existiert.



$$F_B = F_S$$

$$\boxed{F_B = 2F_S \quad (1)}$$

~ Am Ende unklar ob Kanizki



$$\Delta L_B^{(\Delta T)} = \alpha_B L_0 (\Delta T)$$

$$\Delta L_S^{(\Delta T)} = \alpha_S L_0 (\Delta T)$$

$$\Delta L_S^f = \Delta L_B^f \Rightarrow \Delta L_S^{(\Delta T)} + \Delta L_B^{(f)} = \Delta L_B^{(\Delta T)} - \Delta L_B^{(f)}$$

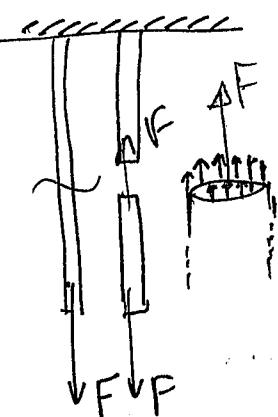
$$\alpha_S L_0 (\Delta T) + \frac{F_S k}{A_S E_S} = \alpha_B L_0 (\Delta T) - \frac{F_B k}{A_B E_B}$$

$$(1) \left\{ \sigma_S = E_S (\alpha_S - \alpha_B) \Delta T \right. \Rightarrow \left[\sigma_S = 1.2 \times 10^6 \Delta T \right] \quad \frac{\sigma_S}{\sigma_B} < \frac{\sigma_{yS}}{\sigma_{yB}}$$

$$(2) \left\{ \sigma_B = 2E_S \frac{A_S}{A_B} (\alpha_B - \alpha_S) \Delta T \right. \Rightarrow \left[\sigma_B = 0.6 \times 10^6 \Delta T \right]$$

$$\Delta T = \sigma_{yB} \rightarrow \dots (\Delta T) = 133.3^{\circ}\text{C}$$

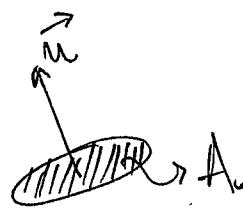
Η έννοια της Διατηγτικής Τάσης



$$\frac{F}{A} = \sigma.$$

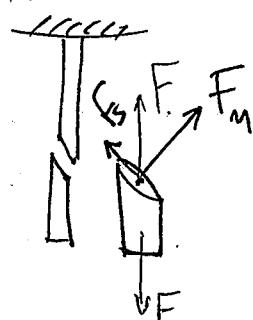
$$\sigma_n = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A}$$

Ορδιν. τάση.



Η τεφρή έχει κάπετα

Αν γίνει άσβη:



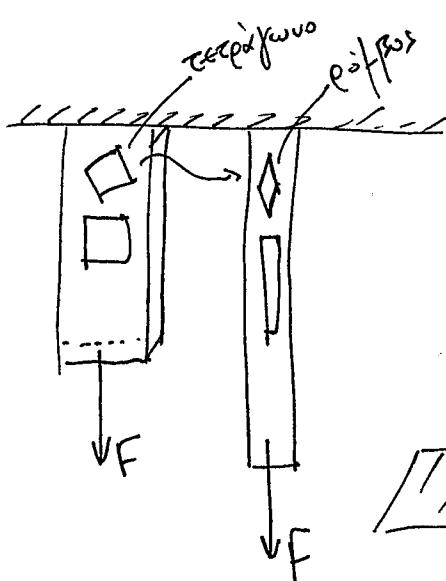
$$\begin{cases} \sigma_n = \frac{F_m}{A} \\ \tau = \frac{F_s}{A} \end{cases}$$

Ορδιν. τάση

$$\varepsilon = \frac{\Delta L}{L_0}$$

Διατηγτική τάση

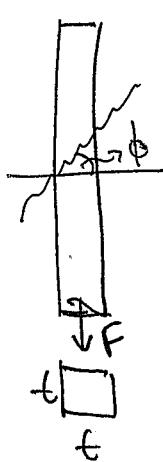
$$\gamma =$$



Διατηγτική Τάση → Η ζύγι
+ τας αρχικως ορδιν. γωνίες.

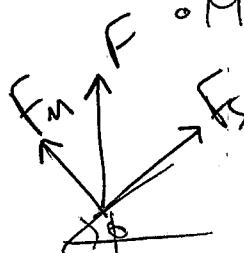
• Με ορδιν. τάση δύος,
ηραπέται Σ και γ

• Με διατηγτική τάση γ .



$$\sigma_n = \sigma_n(\phi)$$

$$\tau = \tau(\phi)$$



$$\sigma_n = \cos \phi F$$

$$\sigma_n = \cos \phi \frac{F}{A}$$

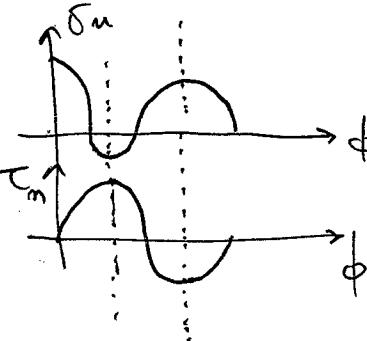
$$\tau = \sin \phi F$$

$$\tau = \sin \phi \frac{F}{A}$$

$$\begin{aligned} \tau &= \cos \phi \frac{F}{A} \\ \delta &= \sin \phi \frac{F}{A} \end{aligned}$$

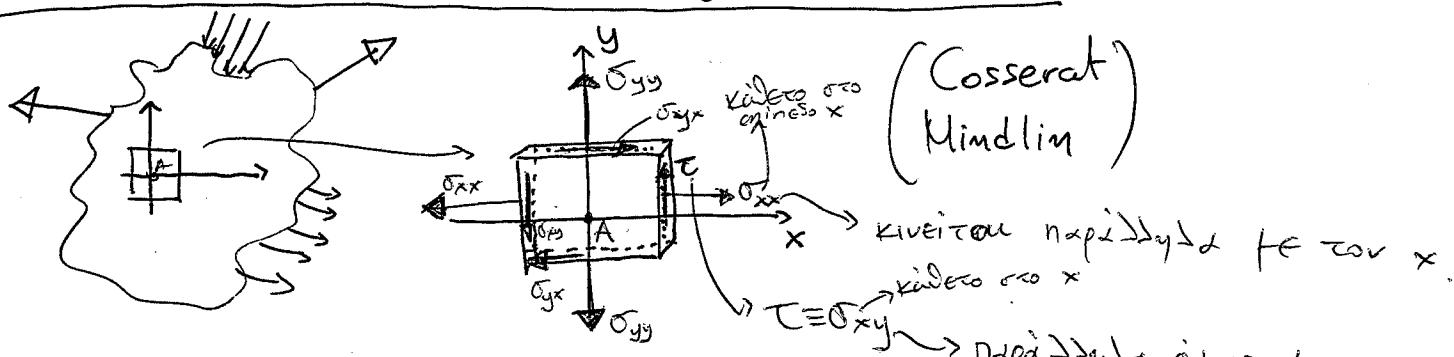
Ιδιαίτερη γωνία ή ζύγιον

$$A = t_x \quad A = t \frac{t}{\cos \phi} \quad \left. \begin{array}{l} \sigma_u = \frac{F_u}{A} = \frac{\sigma_0}{\cos^2 \phi} \\ \sigma_u = \sigma_0 \cos^2 \phi \end{array} \right\} \quad \tau = \frac{F_s}{A} = \frac{F \sin \phi \cos \phi}{A} = \frac{\sigma_0 \sin 2\phi}{2}$$



↔ Τίνει εξωτερική διατήρηση και ορθή.

Ο ΤΑΝΥΣΤΗΣ ΤΗΣ ΤΑΞΕΩΣ 2-D (ΘΗΡΕΥΤΙΚΗ ΜΕΔΙΑ)



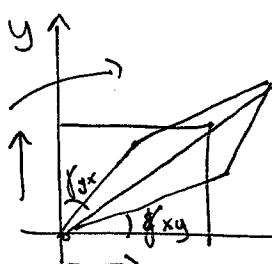
$$\sigma_{yx} = \sigma_{xy}$$

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

$$i,j = x,y$$

$$\epsilon_{ij} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{yx} & \epsilon_{yy} \end{pmatrix}$$

$$\epsilon_{ij} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{yx} & \epsilon_{yy} \end{pmatrix}$$



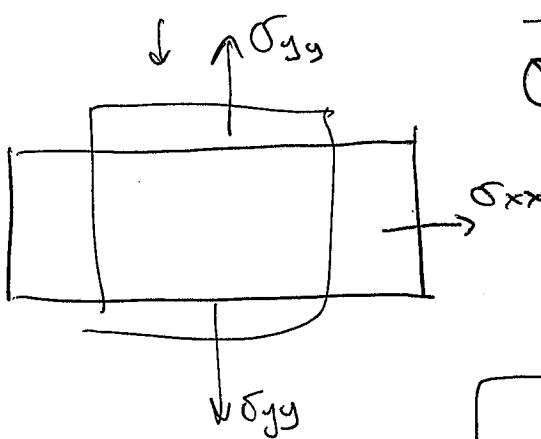
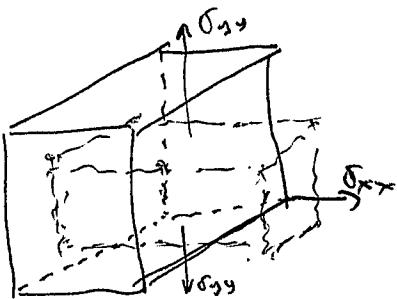
Αυτή η δύο δευτεραριθμητικής ισχύς

$$\gamma = \gamma_{xy} + \gamma_{yx}$$

$$\hookrightarrow \epsilon_{xy} = \epsilon_{yx} = \frac{\gamma_{xy} + \gamma_{yx}}{2}$$

ΟΙ ΚΑΤΑΣΤΑΤΙΚΕΣ ΕΞΙΣΩΣΕΙΣ ΚΑΤΑ Hooke
ΣΕ 3-D ΣΕ ΤΡΑΜΜΙΚΟΣ ΕΛΑΣΤΙΚΑ ΦΟΥΑΤΑ

• Αρχική Ενδιάμεσης



	x	y	z	Tείτος
Aιτία	/ /	/ /	/ /	/ / /
σ_{xx}	$\epsilon_{xx} = \frac{\sigma_{xx}}{E}$	$\epsilon_{yy} = -\nu \frac{\sigma_{xx}}{E}$	$\epsilon_{zz} = -\nu \frac{\sigma_{xx}}{E}$	X X X
σ_{yy}	$\epsilon_{yy} = \frac{\sigma_{yy}}{E}$	$\epsilon_{xx} = -\nu \frac{\sigma_{yy}}{E}$	$\epsilon_{zz} = -\nu \frac{\sigma_{yy}}{E}$	X X X
σ_{zz}	$\epsilon_{zz} = \frac{\sigma_{zz}}{E}$	$\epsilon_{yy} = -\nu \frac{\sigma_{zz}}{E}$	$\epsilon_{xx} = -\nu \frac{\sigma_{zz}}{E}$	X X X

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} + (-\nu \frac{\sigma_{yy}}{E}) + (-\nu \frac{\sigma_{zz}}{E})$$

$$\left[\begin{array}{l} \epsilon_{xx} = \frac{1}{E} [\sigma_{xx} - \nu (\sigma_{yy} + \sigma_{zz})] \\ \vdots \\ \epsilon_{yy} = \frac{1}{E} [\sigma_{yy} - \nu (\sigma_{zz} + \sigma_{xx})] \\ \epsilon_{zz} = \frac{1}{E} [\sigma_{zz} - \nu (\sigma_{yy} + \sigma_{xx})] \end{array} \right]$$

Διατήρηση

$$\rightarrow \left[\begin{array}{l} \sigma_{xy} = f_{xy} G = \sigma_{yx} \\ \sigma_{yz} = f_{yz} G = \sigma_{zy} \\ \sigma_{zx} = f_{zx} G = \sigma_{xz} \end{array} \right] = 2 \epsilon_{xy} G$$

$$\left[\begin{array}{l} \sigma_{yy} = f_{yy} G \\ \sigma_{zz} = f_{zz} G \\ \sigma_{xx} = f_{xx} G \end{array} \right] = 2 \epsilon_{yz} G$$

$$\left[\begin{array}{l} \sigma_{yy} = f_{yy} G \\ \sigma_{zz} = f_{zz} G \\ \sigma_{xx} = f_{xx} G \end{array} \right] = 2 \epsilon_{zx} G$$

$$G = \frac{E}{2(1+\nu)}$$

$$\rightarrow \left\{ \begin{array}{l} \epsilon_{xx} = \frac{1}{E} (\sigma_{xx} - \nu \sigma_{yy}) \\ \epsilon_{yy} = \frac{1}{E} (\sigma_{yy} - \nu \sigma_{xx}) \\ \epsilon_{xy} = \frac{1}{E} \left(\frac{1+\nu}{E} \right) \sigma_{xy} \end{array} \right\}$$

2-D

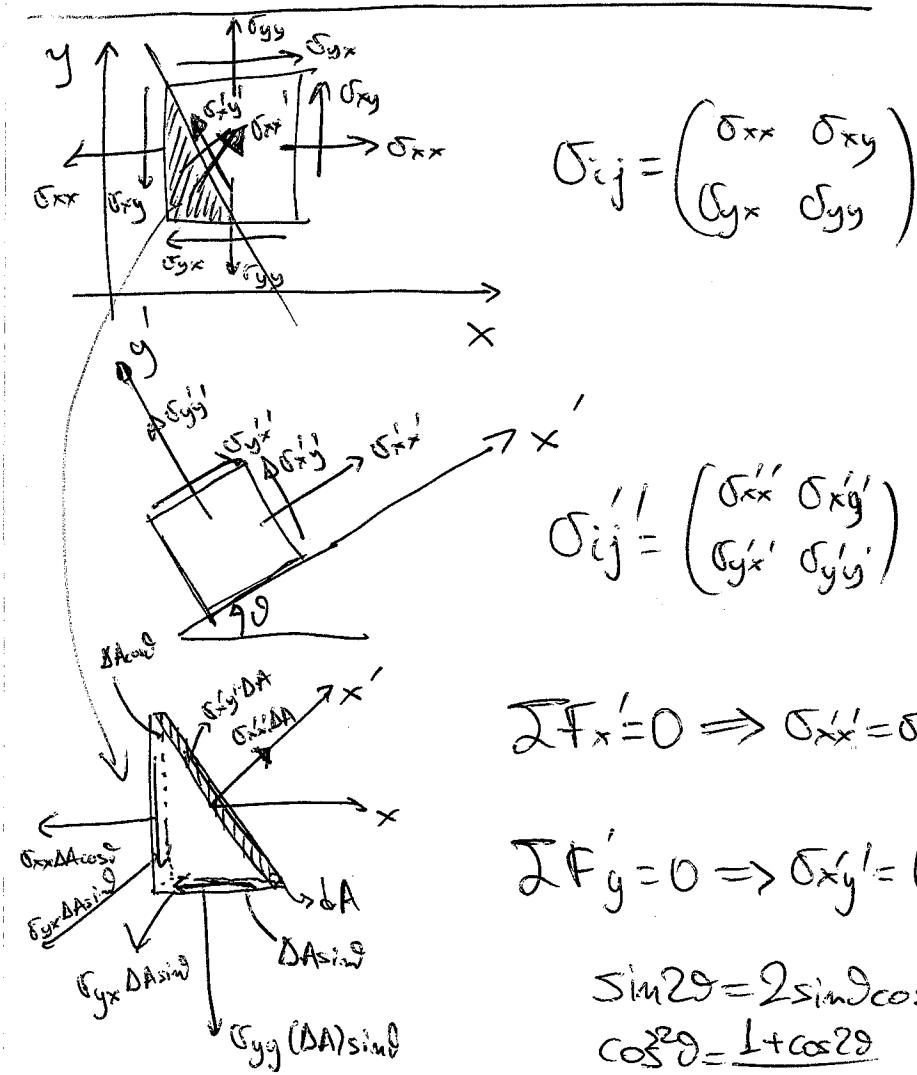
$$\stackrel{1-2}{\Rightarrow} \sigma_{xx} + \sigma_{yy} = \frac{1}{E} \left[(\sigma_{xx} + \sigma_{yy}) - v(\sigma_{xx} - \sigma_{yy}) \right] \quad (3)$$

$$\stackrel{1-2}{\Rightarrow} \sigma_{xx} - \sigma_{yy} = \frac{1}{E} \left[(\sigma_{xx} - \sigma_{yy}) - v(\sigma_{yy} - \sigma_{xx}) \right] \quad (4)$$

$$\stackrel{3+4}{\Rightarrow} 2\sigma_{xx} = \frac{1}{E} \left[(\sigma_{xx} + \sigma_{yy}) - v(\sigma_{xx} + \sigma_{yy}) + (\sigma_{xx} - \sigma_{yy}) + v(\sigma_{xx} - \sigma_{yy}) \right]$$

$$\Rightarrow 2\sigma_{xx} = \frac{1}{E} \left[(\sigma_{xx} + \sigma_{yy})(1-v) + (\sigma_{xx} - \sigma_{yy})(1+v) \right]$$

$$\stackrel{(3-4)}{\Rightarrow} 2\sigma_{yy} = \frac{1}{E} \left[\dots \right]$$



$$\sum F_x' = 0 \Rightarrow \sigma'_{xx}' = \sigma_{xx} \cos^2 \theta + \sigma_{yy} \sin^2 \theta + \sigma_{xy} 2 \sin \theta \cos \theta$$

$$\sum F_y' = 0 \Rightarrow \sigma'_{xy}' = (\sigma_{yy} - \sigma_{xx}) \sin \theta \cos \theta + \sigma_{xy} (\cos^2 \theta - \sin^2 \theta)$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \frac{1 + \cos 2\theta}{2} \quad \sin 2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\boxed{\begin{aligned} \circ \sigma'_{xx}' &= \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\theta + \sigma_{xy} \sin 2\theta \\ \circ \sigma'_{yy}' &= \frac{\sigma_{xx} + \sigma_{yy}}{2} - \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos 2\theta - \sigma_{xy} \sin 2\theta \end{aligned} \quad \circ \sigma'_{xy}' = -\frac{\sigma_{xx} - \sigma_{yy}}{2} \sin 2\theta + \sigma_{xy} \cos 2\theta}$$

① Ακρότατα των ορθών.

$$\hookrightarrow \frac{d\sigma_{xx}}{d\theta} = 0 \Rightarrow \boxed{\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}}$$

Όποιο φέρει την ορθή
υφίστανται σε αυτή τη γενική θ.p.

$$\sin 2\theta_{p1,2} = \pm \frac{\tau_{xy}}{\sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}}$$

$$\cos 2\theta_{p1,2} = \pm \frac{\frac{\sigma_{xx} - \sigma_{yy}}{2}}{\sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}}$$

$$\sigma_{1,2} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$\boxed{\sigma_x' = 0}$$

θ_p .

Κύριο Ενίσθιο → Ηλεγχούμενη ορθή
→ Μηδενικός διεργατικός.

② Ακρότατα των διεργατικών

$$\hookrightarrow \frac{d\sigma_x'}{d\theta} = 0 \rightarrow \boxed{\tan 2\theta_s = \cot 2\theta_p}$$

θ_s shear angle

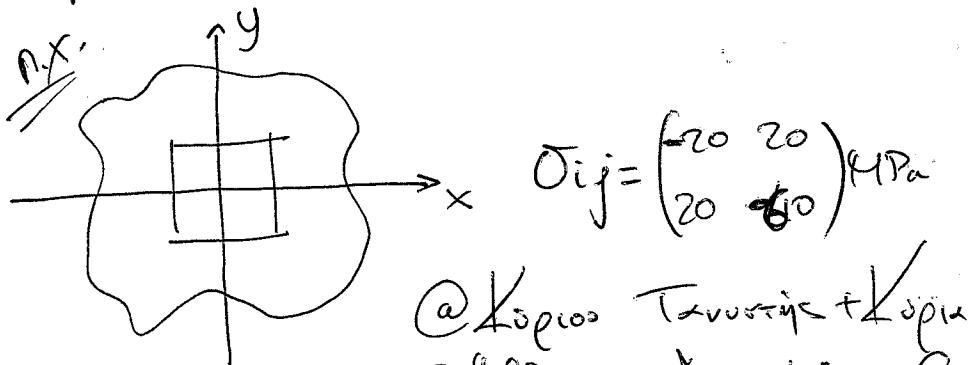
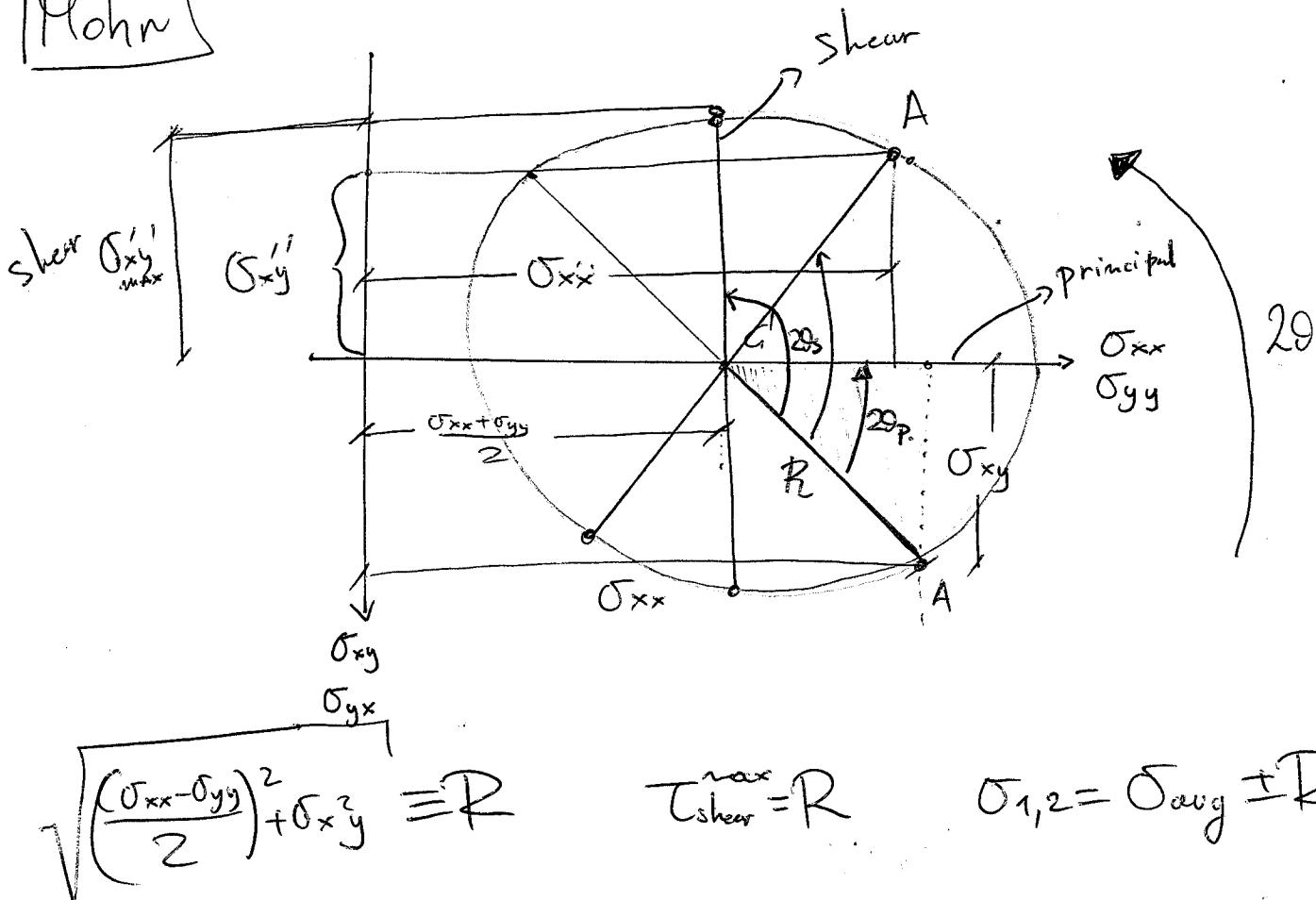
$$T_{shear,max} = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

θ_{shear}

$$\sigma_{avg} = \frac{\sigma_{xx} + \sigma_{yy}}{2}$$



Mohr

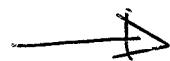


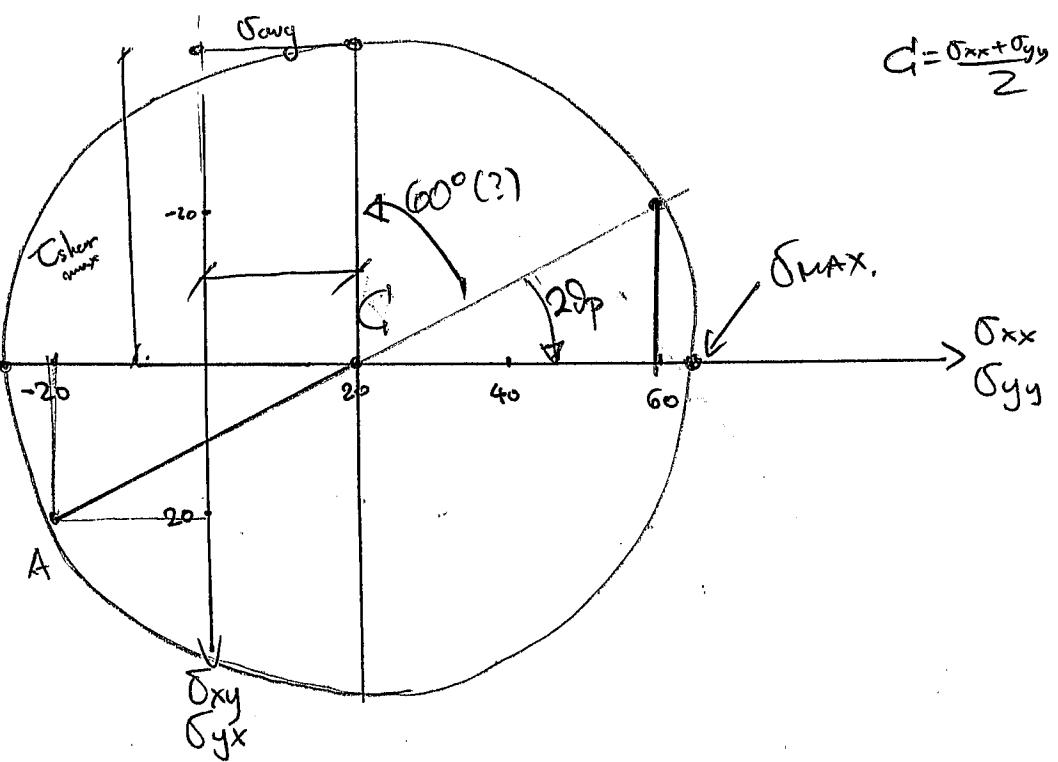
@ Kopios Tavwsijs + Kopis Enineda

(B) Heliotes ΔΙΚΤΙΚΕΣ + ENINEDA ZOUS

(Y) O Tavwsijs zw Taxeww $\theta = 30^\circ$ A.D. O.

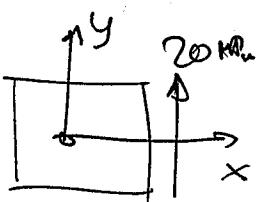
→ Ηλ. Οδοιν : In ovalouzia kan geperia.





$$C = \frac{\sigma_{xx} + \sigma_{yy}}{2}$$

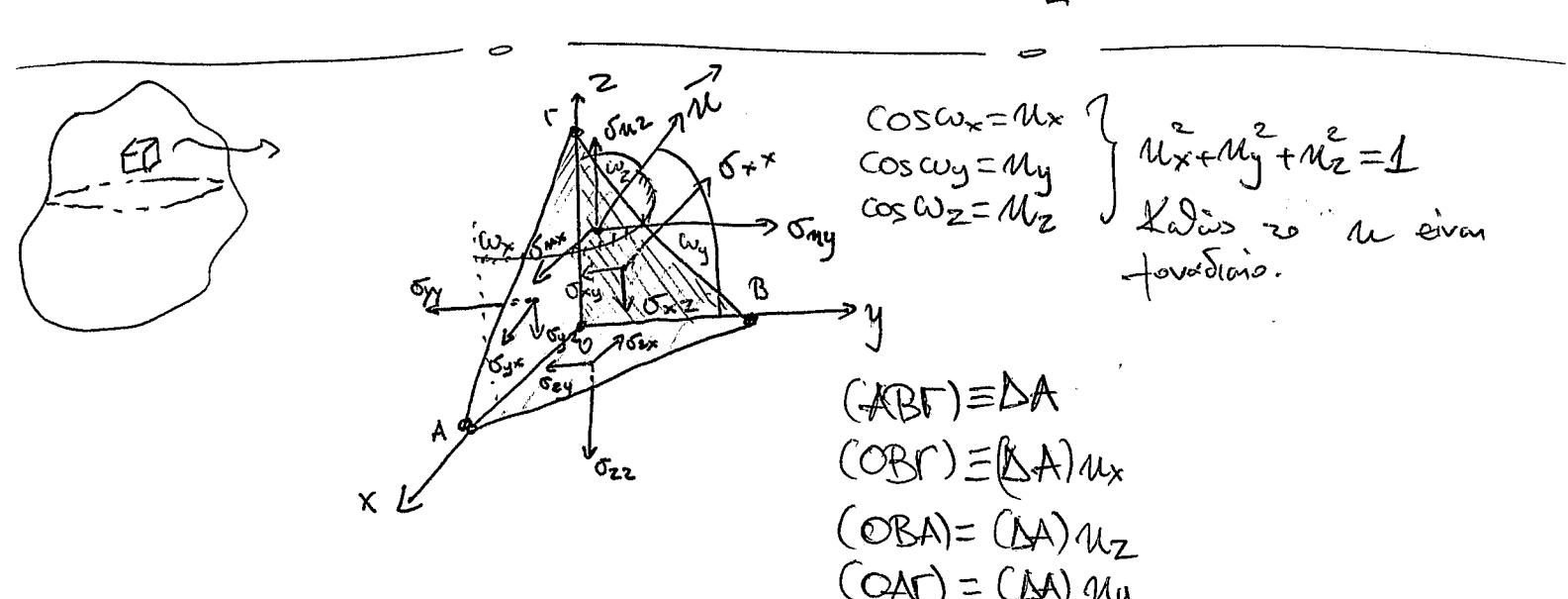
$$\sigma_{ij} = \begin{pmatrix} -20 & 20 \\ 20 & 60 \end{pmatrix}$$



$$\tan 2\theta_p = \frac{\sigma_{xy}}{\frac{\sigma_{xx} - \sigma_{yy}}{2}} = \frac{2 \cdot 20}{-20 - 60} = -\frac{1}{2}$$

$$\tan 2\theta_p = 26.56^\circ \quad \theta_p \approx 13^\circ$$

$\sim 2\text{m}\nu \sigma_{xy}$ bei 30° AD \rightarrow $2\text{m}\nu \sigma_{xy}$ 60°



$$(AB\Gamma) = \Delta A$$

$$(OBR) = (\Delta A) u_x$$

$$(OBA) = (\Delta A) u_z$$

$$(OAR) = (\Delta A) u_y$$

$$(COB)$$

| Zappennid $\sim \sum F_x = 0 \sim \Delta F_{xx} = -\sigma_{xx} (\Delta A) u_x$

$$\Delta F_{zx} = +\sigma_{zx} (\Delta A) u_z$$

$$\Delta F_{yx} = +\sigma_{yx} (\Delta A) u_y$$

$$\Delta F_{ux} = \sigma_{ux} (\Delta A)$$

$$\Delta F_{mass} = F \times \frac{1}{3} (\Delta A) \Delta h$$

Δ Δ φερτού Δετέρας
Τιμής Αγνοίας (καθώς
τον αύξει γίνεται
η πυκνότητα).

L Oι φαβέες διν. Λ
επιθέτων

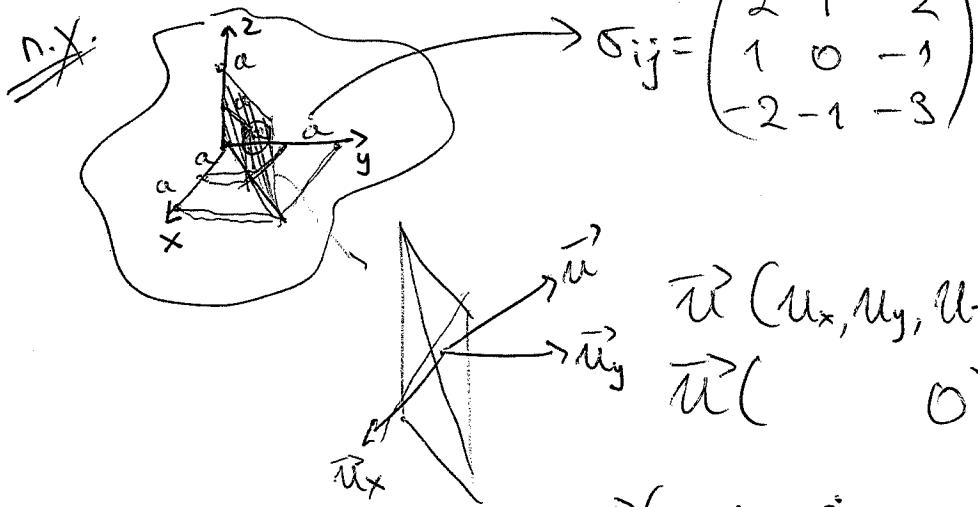
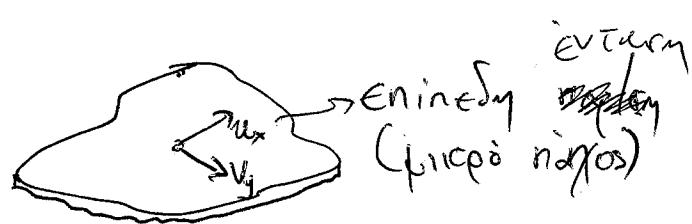
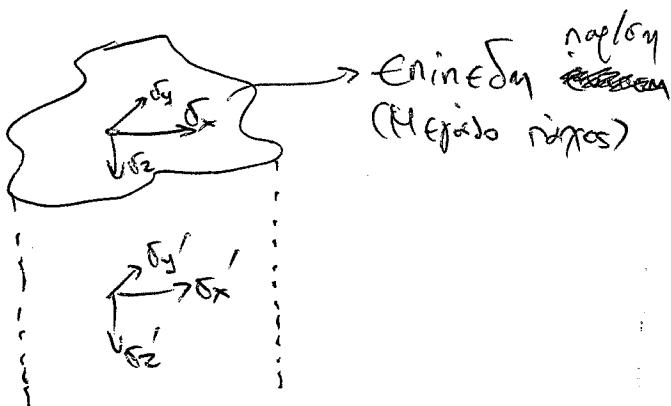
$$\Rightarrow \sigma_{ux}(\Delta A) = \sigma_{xx}(\Delta A)u_x + \sigma_{xy}(\Delta A)u_y + \sigma_{zx}(\Delta A)u_z \Rightarrow$$

$$\begin{aligned}\sigma_{ux} &= \sigma_{xx}u_x + \sigma_{xy}u_y + \sigma_{xz}u_z \\ \sigma_{uy} &= \sigma_{xy}u_x + \sigma_{yy}u_y + \sigma_{yz}u_z \\ \sigma_{uz} &= \sigma_{xz}u_x + \sigma_{yz}u_y + \sigma_{zz}u_z\end{aligned}\left.\right\} \rightarrow \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix} \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = (\sigma_{ux}, \sigma_{uy}, \sigma_{uz})$$

$\Rightarrow \boxed{\vec{\sigma}_u = \sigma_{ij} \vec{n}}$

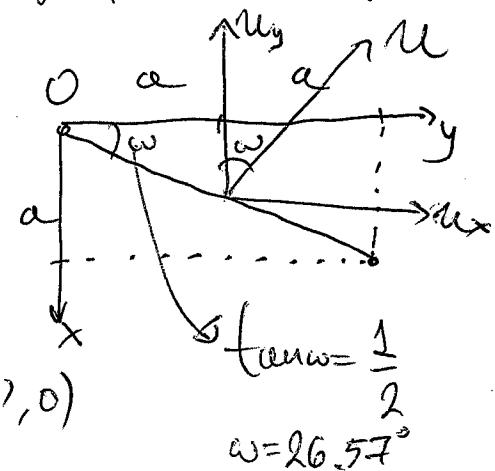
$$\rightsquigarrow \vec{\sigma}_{ut} \vec{n}_{ut} \Rightarrow (\vec{\sigma}_u \cdot \vec{n}) \vec{n} = \vec{\sigma}_{ut}$$

$$\rightarrow \rightarrow \vec{\sigma}_{ut} = \vec{\sigma}_{ut}$$

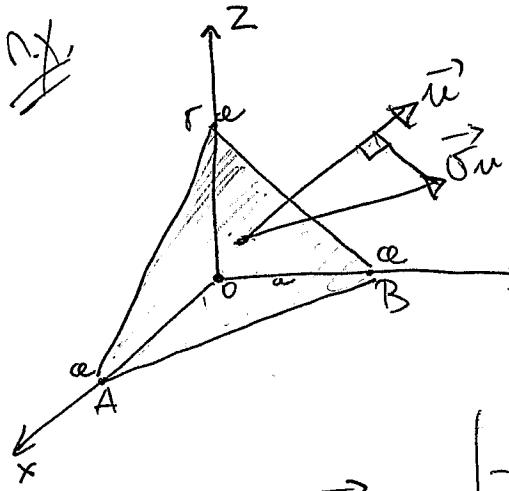


$$\vec{\sigma}_{ij} = \begin{pmatrix} 2 & 1 & -2 \\ 1 & 0 & -1 \\ -2 & -1 & -3 \end{pmatrix}$$

Ηα εργαζεται διανυσματικης μορφης στην επιφανεια.



$$\therefore \vec{\sigma}_u = \begin{pmatrix} 2 & 1 & -2 \\ 1 & 0 & -1 \\ -2 & -1 & -3 \end{pmatrix} \begin{pmatrix} -0.89 \\ 0.45 \\ 0 \end{pmatrix}$$



$$\sigma_{ij} = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & -1 & -2 \end{pmatrix} \text{ MPa} \quad (\text{In } O)$$

Ωε εργαί γερή τιον νω σπέ

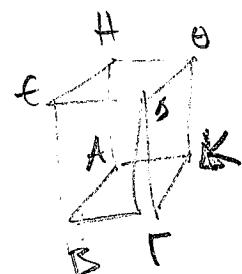
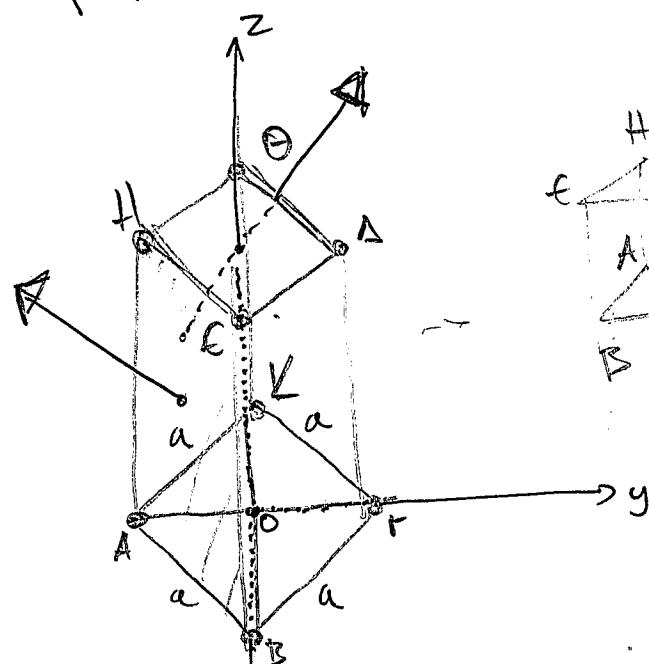
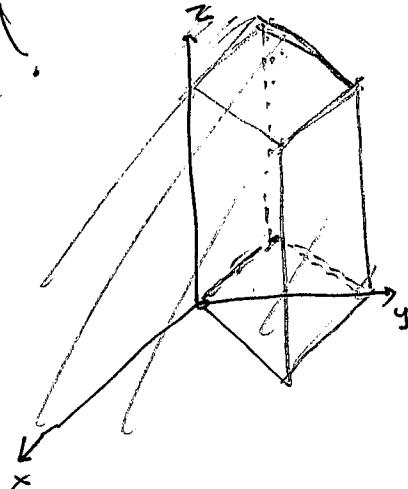
ενιέστο ΑΒΓ.

$$\vec{n} = \frac{\vec{AB} \otimes \vec{AC}}{|\vec{AB} \otimes \vec{AC}|} = \frac{\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 0 \\ -1 & 0 & 1 \end{vmatrix} |a|^2}{|\vec{n}|} = \frac{2\vec{i}\vec{c} + \vec{j}\vec{f} + 2\vec{k}\vec{e}}{a\sqrt{2^2+1+2^2}} \rightarrow \vec{n} = \frac{1}{3}(2\vec{i} + \vec{j} + 2\vec{k})$$

$$\vec{\sigma}_n = \sigma_{ij} \vec{n} = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & -1 & -2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \frac{1}{3} \rightarrow \vec{\sigma}_{n\alpha} = \left(\frac{10}{3}, 0, -1 \right) \text{ MPa}$$

$$\hookrightarrow \vec{\sigma}_{nn} = (\vec{\sigma}_n \cdot \vec{n}) \vec{n}$$

✓ P.X!



Διωρφα τιρη

$$\rightarrow (\vec{ABC}H) = (2, 1, -1) \text{ MPa} \times t \rightarrow (2t, t, -t)$$

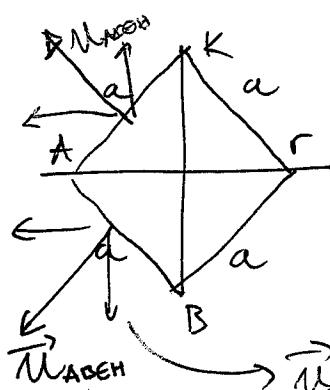
$$\rightarrow (\vec{AK\Theta}H) = (-1, 0, 2) \text{ MPa} \times t \rightarrow (-t, 0, 2t)$$

$$\rightarrow |(\vec{AE\Delta\Theta})| = 2 \text{ MPa}$$

Πλας ο τενερή:

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$

→ Fließdauer:

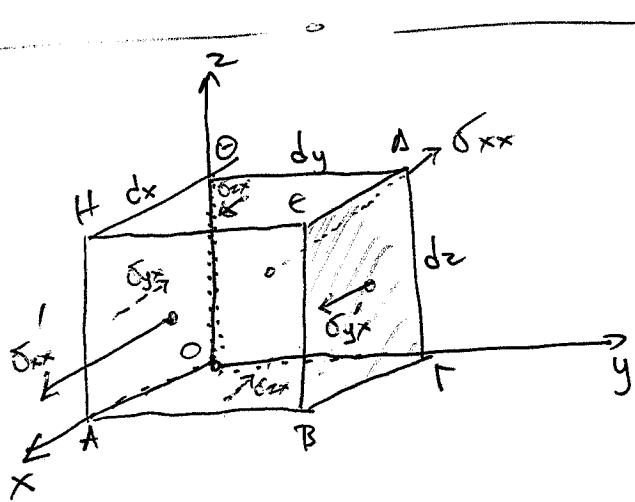


$$\vec{n}_{ABEH} \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0 \right) \Rightarrow$$

$$\Rightarrow \frac{\sqrt{2}}{2} \sigma_{xx} - \frac{\sqrt{2}}{2} \sigma_{xy} + 0 = (2t, 1t, -1t)$$

→ To id. + fass. ACHH ...

→ To t. reichen an → fero zu σ_{xx}/σ_{yy}



$$\sum f_x = 0$$

$$\sigma'_{xx} = \sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx + \dots$$

$$(\sigma'_{xx} - \sigma_{xx}) dy dz + (\sigma_yx - \sigma_{yx}) dx dz + (\sigma_{zx} - \sigma_{xz}) dx dy + f_x dx dy dz = 0$$

$$\Rightarrow \left(\sigma_{xx} + \frac{\partial \sigma_{xx}}{\partial x} dx - \sigma_{xx} \right) dy dz + \left(\sigma_{yx} + \frac{\partial \sigma_{yx}}{\partial y} dy - \sigma_{yx} \right) dx dz + \left(\sigma_{zx} + \frac{\partial \sigma_{zx}}{\partial z} dz - \sigma_{zx} \right) dx dy + f_x dx dy dz = 0$$

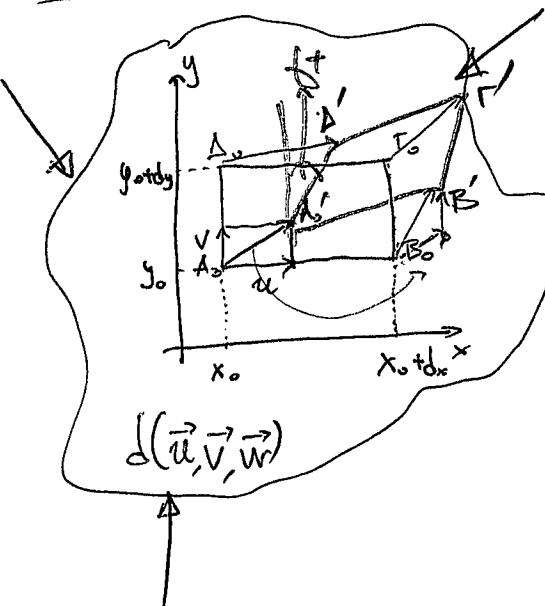
$$\sim \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \sigma_{zx}}{\partial z} + f_x = 0 \quad (1)$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + f_y = 0 \quad (2)$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + f_z = 0 \quad (3)$$

TANYATHTI ΔAP/ΣΕΩΝ - ΣΧΕΣΙΣ ΗΕΤΑΠΟΝΙΣΕΩΝ

ΔAP/ΣΕΩΝ



$$A(x, y)$$

$$B_0(x + dx_0, y)$$

$$C_0(x + dx_0, y + dy_0)$$

$$D_0(x, y + dy_0)$$

$$A(x+u, y+v)$$

$$B(x+dx+u+\frac{\partial u}{\partial x}dx, y+v+\frac{\partial v}{\partial x}dx)$$

DEFORMATION
TO A POINT
AND ANG.

$$\Delta(x+u+\frac{\partial u}{\partial y}dy, y+dy+v+\frac{\partial v}{\partial y}dy)$$

$$f(x+dx+u+\frac{\partial u}{\partial x}dx, y+dy+v+\frac{\partial v}{\partial x}dy)$$

$$r(x+dx+u+\frac{\partial u}{\partial x}dx+\frac{\partial u}{\partial y}dy, y+dy+v+\frac{\partial v}{\partial x}dx+\frac{\partial v}{\partial y}dy)$$

$$\epsilon_{xx} = \frac{\Delta L_x}{L_0} = \frac{AB - A_0 B_0}{L_0} = \frac{1}{dx + du/dx} \rightarrow$$

$$\rightarrow = \frac{\sqrt{(dx + \frac{\partial u}{\partial x}dx)^2 + (\frac{\partial v}{\partial x}dx)^2}}{dx} = \frac{\sqrt{(dx + (\frac{\partial u}{\partial x})^2 dx^2 + 2(\frac{\partial u}{\partial x})dx^2 + (\frac{\partial v}{\partial x})^2 dx^2)}}{dx}$$

$$\rightarrow = dx \sqrt{1 + (\frac{\partial u}{\partial x})^2 + 2\frac{\partial u}{\partial x}} \rightarrow \text{approx if } \frac{\partial u}{\partial x} \ll 1$$

$$\rightarrow = dx \sqrt{1 + 2\frac{\partial u}{\partial x}} = \sqrt{1 + 2\frac{\partial u}{\partial x} - \frac{1}{4}\frac{\partial u}{\partial x}^2}$$

$$\boxed{\epsilon_{xx} = \frac{\partial u}{\partial x}}$$

$$\rightarrow \text{also } \boxed{\epsilon_{yy} = \frac{\partial v}{\partial y}}$$

$$\boxed{\epsilon_{zz} = \frac{\partial w}{\partial z}}$$

$$f_x \approx \text{true } f_x = \frac{\frac{\partial V}{\partial x} f_x}{f_x + \frac{\partial u}{\partial x} f_x} = \frac{\frac{\partial v}{\partial x}}{1 + \frac{\partial u}{\partial x}} \approx \frac{\partial v}{\partial x}$$

$$f_y = \frac{\partial u}{\partial y}$$

$$f_x + f_y = 2\varepsilon_{xy} = 2\varepsilon_{yx} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \Rightarrow \varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$\left(\varepsilon_{xx} = \frac{1}{2} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} \right) \right) \quad \left(\varepsilon_{yz} = \frac{1}{2} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \right)$$

Kontinuierliche Medien Theorie

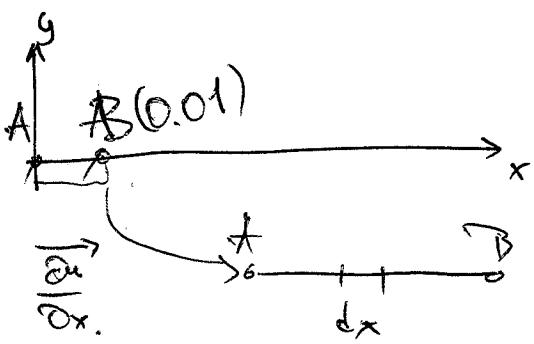
$$(u) \rightarrow (\varepsilon_{ij}) \xrightarrow{\text{Hooke}} (\sigma_{ij}) \rightarrow F_i$$

$$\textcircled{6} \rightarrow \textcircled{1} \quad E = 200 \text{ GPa} \quad v = 0.3 \quad \vec{\alpha} = [(2x^2 + 3y)\vec{i} + (3y^2 + 2xy)\vec{j}] \times 10^{-5} \text{ m}$$

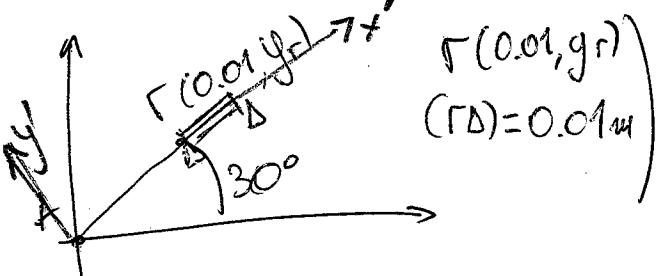
$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = 4x, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} = 6y + 2x, \quad \varepsilon_{xy} = \frac{1}{2} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = \frac{1}{2} (3 + 2y)$$

$$\varepsilon_{ij} = \begin{pmatrix} 4x & \frac{1}{2}(3+2y) \\ \frac{1}{2}(3+2y) & 6y + 2x \end{pmatrix} \times 10^{-5}$$

$$\frac{\partial x}{\partial x} + 1 = 1 + 0.01 \neq 0.4 \times 10^2 / 10^2 = 4 \text{ kN}$$



$$d(\Delta L) = \varepsilon_{xx}(x) dx = \int \varepsilon_{xx} dx = 4 \frac{x^2}{2} \Big|_0^{0.01} = d(0.01)$$



$$\varepsilon'_{xx} = \frac{1}{2} (\varepsilon_{xx} + \varepsilon_{yy}) + \frac{1}{2} (\varepsilon_{xx} - \varepsilon_{yy}) \cos 2\theta + \varepsilon_{xy} \sin 2\theta$$

$$\varepsilon'_{xx} = 3(x+y) + \frac{2x-6y}{2} \frac{1}{2} + \left(\frac{3}{2} + y \right) \frac{\sqrt{3}}{2} \frac{y}{x} \quad \frac{y}{x} = \frac{\sqrt{3}}{3}$$

$$\varepsilon'_{xx}(x) \sim \int dx' \sim \frac{dx'}{dx} = \tan 30^\circ$$

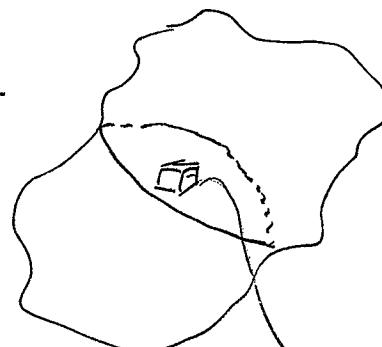
10 → 6, 7.

(27)

Kριτήρια Αστοχίας (Uno σταύρωσης συνδικάς)

(I) Von Mises

~~Σταύρωσης~~

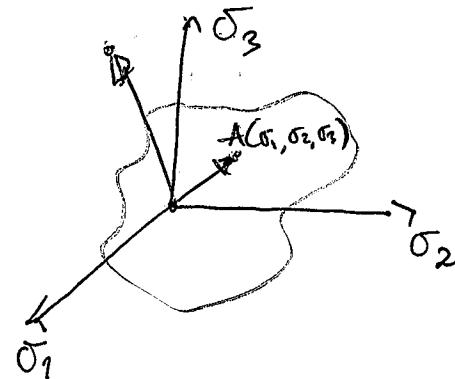


$$\rightarrow \sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix} ? \quad \sigma_? < \sigma_{cr} ?$$

$$H_1(\sigma_{ij}, \epsilon_{ij}, \chi_{ij}, T, \sigma_{j,0}) \times$$

$$H_2(\sigma_{ij}) \text{ μερική φόρμα \ } \sigma_{ij}(T)$$

$$H_1(\sigma_I, \sigma_{II}, \sigma_{III})$$



$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz} = \sigma_I + \sigma_{II} + \sigma_{III}$$

$$P = \frac{I_1}{3}$$

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \rightsquigarrow \begin{pmatrix} \sigma_I & 0 & 0 \\ 0 & \sigma_{II} & 0 \\ 0 & 0 & \sigma_{III} \end{pmatrix}$$

$$\begin{pmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{pmatrix} + \begin{pmatrix} \sigma_{xx}-P & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy}-P & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz}-P \end{pmatrix}$$

deviatoric tensor

no diagonal

$\sqrt{\delta_{\text{primitiv}}} \rightarrow \Delta \text{ stress.}$

$$\Rightarrow H_4(I_1, \sigma_{II}, \sigma_{III})$$

$$H_5(J_2, J_3) = C_5$$

• Όλη η Τίτα

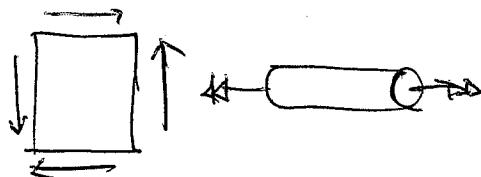
$$\begin{aligned} \text{f}_6(J_2) = C_6 \\ J_2 = C^2 \end{aligned} \quad \rightarrow \sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2 = \sigma_{xx}\sigma_{yy} - \sigma_{yy}\sigma_{zz} - \sigma_{zz}\sigma_{xx} + \\ 3\sigma_{xy}^2 + 3\sigma_{yz}^2 + 3\sigma_{zx}^2 = \sigma_y^2 \text{ yield.}$$

→ Η σύνθεση των επιρρεον στανης και διεύρυνσης αναλογίαν του ανοκτίσιμου τανόσου των τάσεων, έχει την κρίτη ρήσης που είναι ότι τον τάση διαρροή σε τονολόγιο εφαρμοζεται.

$$\sigma_I^2 + \sigma_{II}^2 + \sigma_{III}^2 - \sigma_I \sigma_{II} - \sigma_{II} \sigma_{III} - \sigma_{III} \sigma_I = \sigma_y^2 \quad \text{Κύριο 3-D}$$

$$\begin{array}{l} \xrightarrow{\text{Εγώ}} \left. \begin{array}{l} \sigma_{xx}^2 + \sigma_{yy}^2 - \sigma_{xy}\sigma_{yy} + 3\sigma_{xy}^2 = \sigma_y^2 \\ \sigma_I^2 + \sigma_{II}^2 - \sigma_I \sigma_{II} = \sigma_y^2 \end{array} \right\} 2-D \\ \xrightarrow{\text{Εγώ}} \left. \begin{array}{l} \sigma_{xy}^2 \\ \sigma_I^2 + \sigma_{II}^2 - \sigma_I \sigma_{II} = \sigma_y^2 \end{array} \right\} 2-D \end{array}$$

Η ίδια προσδιορίζει και το κρίτη του Mises, ή σε μερικές εργασίες.



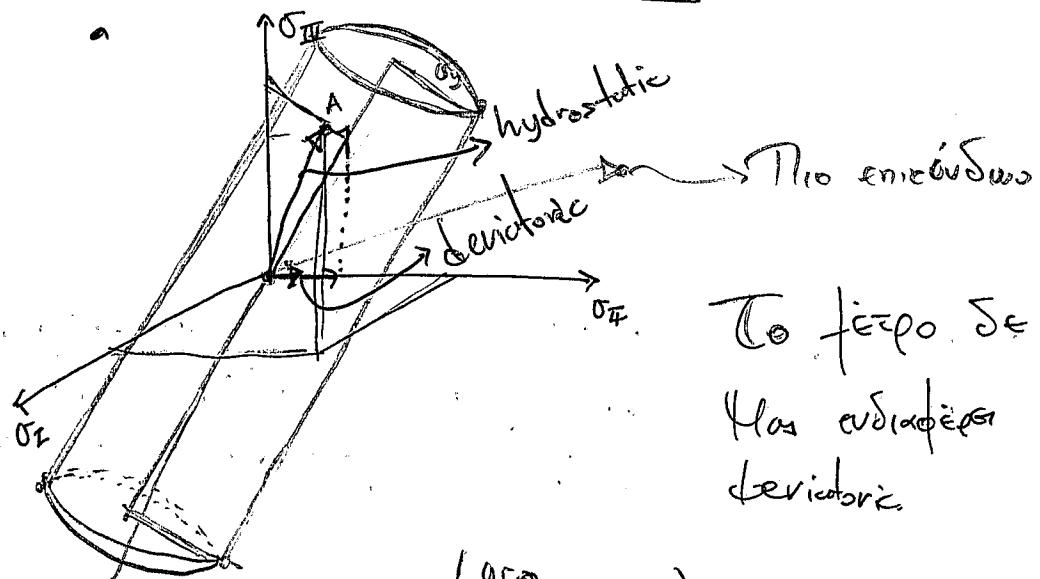
$$\sigma_{ij} = \begin{pmatrix} 0 & \sigma_{xy} & 0 \\ \sigma_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow 3\sigma_{xy}^2 = \sigma_y^2$$

$$3\sigma_y^2 = \sigma_y^2$$

$$\boxed{\sigma_y = \sqrt{3}\sigma_z}$$



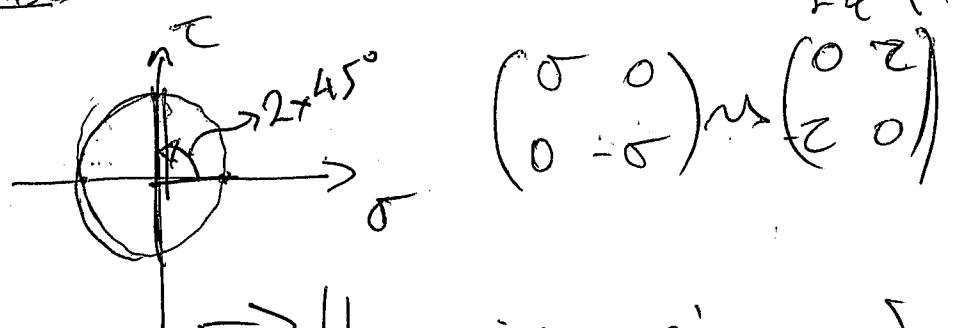
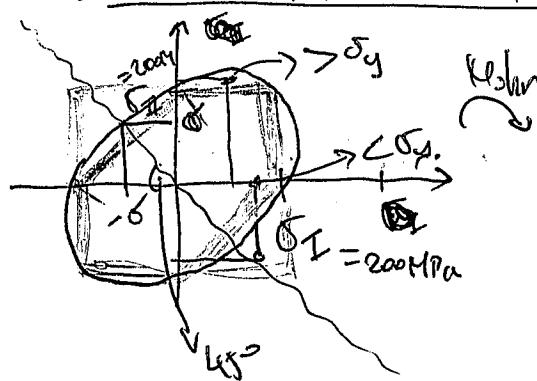
Tensile stress Axial symmetry von Mises



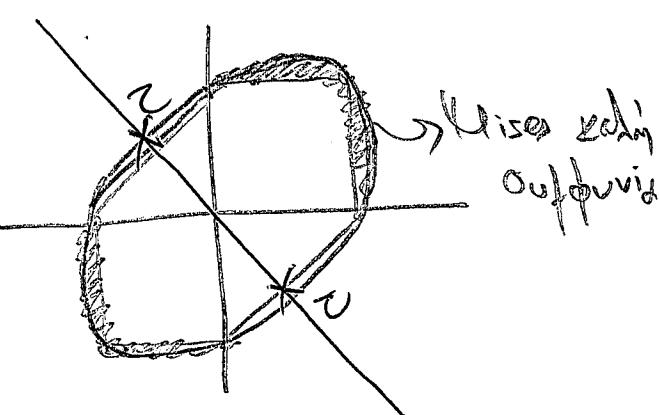
$$\begin{pmatrix} 950 \\ 1000 \\ 1050 \end{pmatrix} \text{ MPa} \quad \begin{pmatrix} 3 \\ 0 \\ 7 \end{pmatrix} \text{ MPa.}$$

$$G \rightarrow P = ?$$

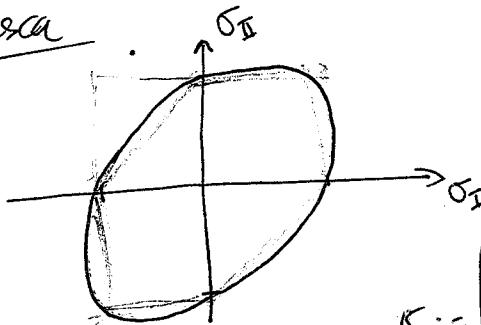
2 → Ειδική ανάλυση von Mises



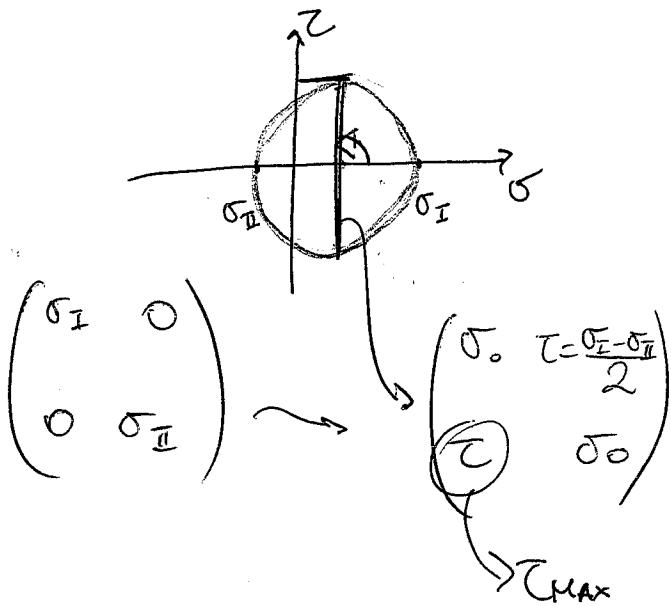
\Rightarrow H στρέψη είναι στα μεταξύ
εφελκυστικού και διατηγμένου
ενισχύσης 45° .



Tresca

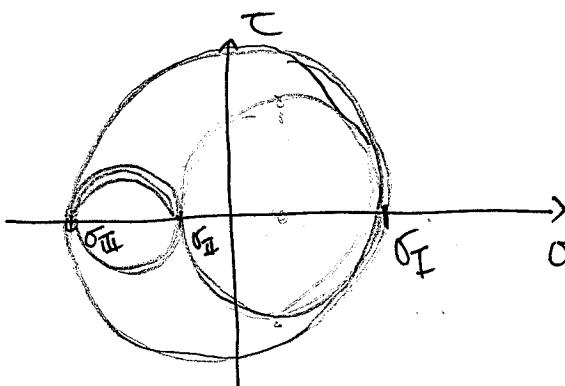


$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \sim$$



\Rightarrow Ασφαλίς οι ενεργές, όταν $\tau_{max} > C$

3-D



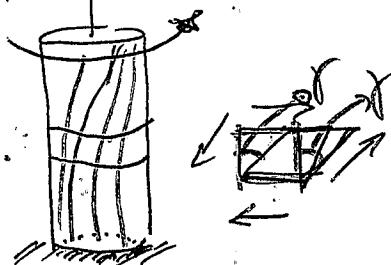
$$\sim \max \left\{ \frac{\sigma_I - \sigma_{II}}{2}, \frac{\sigma_{II} - \sigma_{III}}{2}, \frac{\sigma_{III} - \sigma_I}{2} \right\}$$

$$A \leq C$$

- Προβλήμα ∇ λιξ → ~~hydrostatic~~ ενώ δίνων ενέργεια
- Προβλήμα Tresca → Αν τις εργασίες γιαν διαφορική "εξαπλική", οι ίδιες θα δεν ευθείες ενώ δίνων ενέργεια.



ZΤΡΕΥΜ ΜΗ



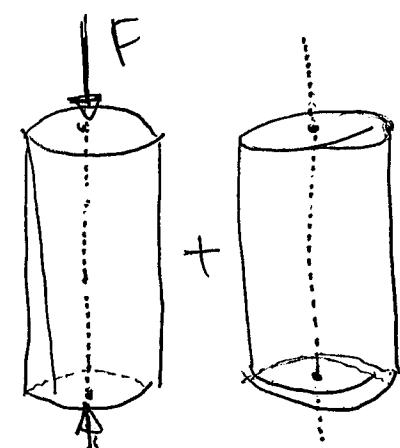
$$\sim \sigma_{ij} = \begin{pmatrix} 0 & \tau_0 & 0 \\ \tau_0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\approx \frac{4t}{I_p} r$$

$$I_p = \iint r^2 dA = I_{xx} + I_{yy}$$

$$I_p^{\text{circular}} = \frac{\pi R^4}{2}$$

→ Διαδικαγμένης ακτίνης $r=4\text{cm}$ είναι κατακευστήματος στο μετρητό γάλου, έτσι $\sigma_y = 400\text{kPa}$. Η πάσχας οποιασδήποτε σε σφουγκλή θετική ή καραβητική ποσηί $Ht=2\text{KN.m}$. Η υλοποίηση της στοιχείωσης γίνεται με στρώση τρύπας και τρέσας.



→ Ενδιδαχτικό

→ Βρίσκω τους ενιπόσας ταυτότητες

$$\left(I_p = \frac{\pi R^4}{2} = 1256 \times 10^{-8} \right)$$

$$\rightarrow \sigma_c = \frac{F}{A} \rightarrow \sigma_c = -0.196 \times 10^4 F \Rightarrow \begin{pmatrix} -0.196 F \times 10^4 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A = \pi r^2 \sim \frac{1}{A} = 0.196 \times 10^4$$

$$\tau = \frac{Ht r}{I_p} = \frac{2.2 \times 10^3}{1256} r \quad + \quad \begin{pmatrix} 0 & 4.97r \\ 4.97r & 0 \end{pmatrix} \text{ MPa.}$$

$$\Rightarrow \sigma_{ij} = \left[\begin{pmatrix} -0.196 F_x \times 10^{-2} & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 497_r \\ 497_r & 0 \end{pmatrix} \right] \text{N/mm}^2$$

$$\sigma_{ij} = \begin{pmatrix} -0.196 F_x \times 10^{-2} & 497_r \\ 497_r & 0 \end{pmatrix} \text{N/mm}^2 \Rightarrow \text{Octahedral stress condition.}$$

$$\Rightarrow \sigma_{ij_{cr}} = \begin{pmatrix} -0.196 F_x \times 10^{-2} & 497R \\ 497R & 0 \end{pmatrix} \rightsquigarrow$$

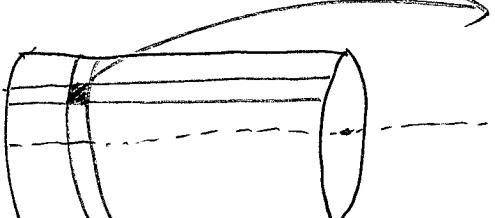
Using $(0.196 F_x \times 10^{-2})^2 + 3(497R)^2 = \left(\frac{\sigma_y}{SF}\right)^2$

$$F = \left[\left(\frac{400}{2} \right)^2 - 3 \cdot 497^2 \cdot R^2 \right] \frac{1}{(0.196 \times 10^{-2})^2} \right]^{1/2}$$

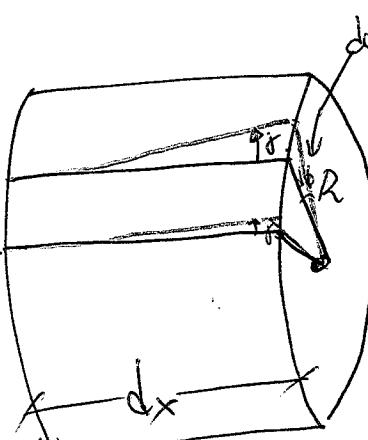
Tresca \rightarrow Kuppli criterion $\rightarrow \begin{pmatrix} \sigma_I & 0 \\ 0 & \sigma_{II} \end{pmatrix} \left(\frac{\sigma_I - \sigma_{II}}{2} \leq \tau_y \right)$

τ_y is yield stress
 $\tau_y = 400 \text{ MPa}$

Anisotropy (now I understand)



L



$$\text{Shear } \gamma = \frac{dS}{dx} = \frac{R \Delta \phi}{dx} = R \dot{\phi}$$

$$\text{Hooke's Law } \tau = G \gamma$$

$$G = R \dot{\phi}$$



$$\text{dM} = \tau dA r = \rho g r dA \quad \begin{array}{l} \text{densidade} \\ \text{massa} \end{array} \rightarrow \tau = \rho g r dA$$

~~$\int \int \int \int \int$~~

$$M = \int \int \int \int \int dM = \int \int \int \int \int \tau dA = \rho g r \int \int \int \int \int dA$$

$$\rightarrow M = \iint_A dM = \rho g \iint_A r^2 dA \rightarrow \boxed{\frac{M}{\rho g} = r^2 I_p}$$

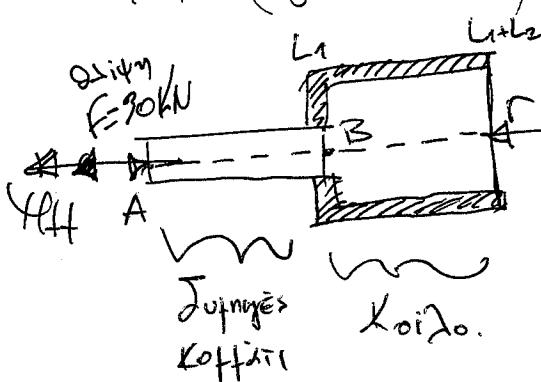
$\tau = \rho g$

$$\frac{M}{\rho g} = \frac{I_p}{r^2} \rightarrow$$

$$r = \boxed{\frac{M}{\rho g I_p}} \rightarrow \boxed{r = \frac{M}{\rho g I_p}}$$

Π.Χ.

Στρέψη ($\sigma_y = 200 \text{ MPa}$)



@ X παρος

(B) παρος

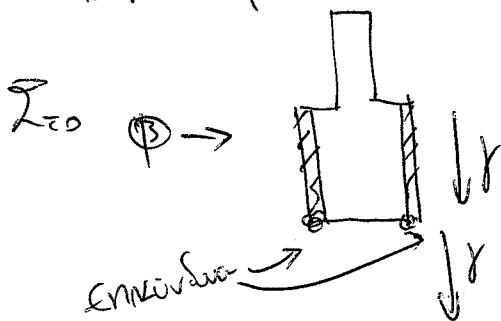


FGDintim.

$\rightarrow AB \rightarrow \Sigma$ πας την ταυταρική φόρμου, οπως η σύγχρονη είναι
Ισονίδια υδροχοήσεων

\rightarrow Στρέψη, κινδυνεών ο ταυδιός του κυλινδρού.

BΓ → Οποιως.



$$AB \rightarrow \sigma_{ij} = \begin{pmatrix} -\frac{F}{A} & \frac{MFR}{I_{P1}} \\ \frac{MFR}{I_{P1}} & 0 \end{pmatrix} \quad R (\text{εφύρωση ταυδιού})$$

$A_2 = \text{επόδιον διεύθυνση}$

$$B\Gamma \rightarrow \sigma_{ij} = \begin{pmatrix} -\frac{F}{A_2} & \frac{MFR_{\text{out}}}{I_{P2}} \\ \frac{MFR_{\text{out}}}{I_{P2}} & 0 \end{pmatrix}$$

$$I_{P1} = \frac{nR^4}{2}$$

$$I_{P2} = \frac{\pi R_{\text{out}}^4}{2} - \frac{\pi R_{\text{in}}^4}{2}$$

(35)

→ Energy Dev principle van principle eccles zuv rigo zwervig,
Tressen.

→ Mises!

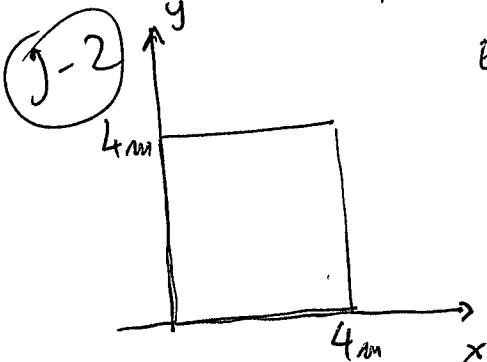
$$\sigma_{xx}^2 + 3\sigma_{xy}^2 \leq \sigma_y^2 \Rightarrow \left(-\frac{F}{A_1}\right)^2 + 3\left(\frac{M+R}{I_{P_1}}\right)^2 \leq (200)^2 \quad (AB)$$

$$\Rightarrow \left(-\frac{F}{A_2}\right)^2 + 3\left(\frac{M+R_{out}}{I_{P_2}}\right)^2 \leq (\sigma_y)^2 \quad (B)$$

AB → ... M_{t_1}

BΓ → ... M_{t_2} $\max M_t = \min(M_{t_1}, M_{t_2})$

→ Lösungs u, ε.



$$E = 0.9375 \text{ GPa}, v = 0.25$$

$$u = (-x + x^2y) \times 10^{-3} \text{ m} \quad \frac{\partial u}{\partial x} = (-1 + 2xy) \times 10^{-3} = \epsilon_{xx}$$

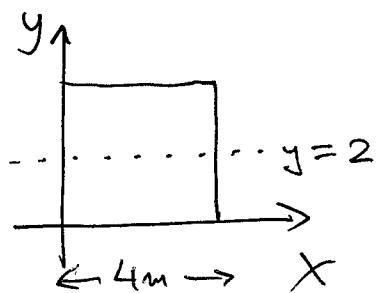
$$v = (2y^2 - \frac{y^3}{3}) \times 10^{-3} \text{ m} \quad \frac{\partial v}{\partial y} = (4y - y^2) \times 10^{-3} = \epsilon_{yy}$$

$$\begin{cases} \frac{\partial u}{\partial y} = x^2 \times 10^{-3} \\ \frac{\partial v}{\partial x} = 0. \end{cases} \quad \left. \begin{array}{l} \rightarrow \epsilon_{xy} = \frac{x^2}{2} \times 10^{-3} \end{array} \right\}$$

$$\epsilon_{ij} = \begin{pmatrix} -1 + 2xy & \frac{x^2}{2} \\ \frac{x^2}{2} & 4y(1-y) \end{pmatrix} \times 10^{-3}.$$

(36)

$$\rightarrow \frac{\partial \varepsilon_{yy}}{\partial y} = 4 - 2y = 0 \Rightarrow \boxed{y=2}$$



(3) $\int_0^4 \varepsilon_{xx} dx + \lambda(x_x + x_y) \Big|_0^4 = -4 + 16 \quad \text{with } \varepsilon_{xx} = 2xy - 1$

$$\rightarrow y=2 \rightarrow \varepsilon_{xx} = (4x - 1) \times 10^{-3}$$

$$\Delta L = \varepsilon L_0 \rightarrow \Delta L_{\text{total}} = \int_0^4 \varepsilon_{xx} dx$$

$$\Delta L_{\text{total}} = \int_0^4 (2x^2 - x) \times 10^{-3} dx$$

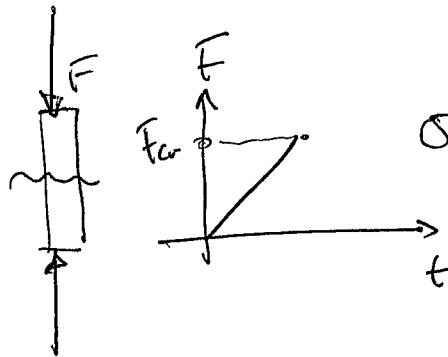
$$\Delta L_{\text{total}} = (2 \times 16 - 4) \times 10^{-3}$$

$$\Delta L_{\text{total}} = 28 \text{ mm}$$

$$\rightarrow [L_{\text{final}} = 4.028 \text{ m}]$$

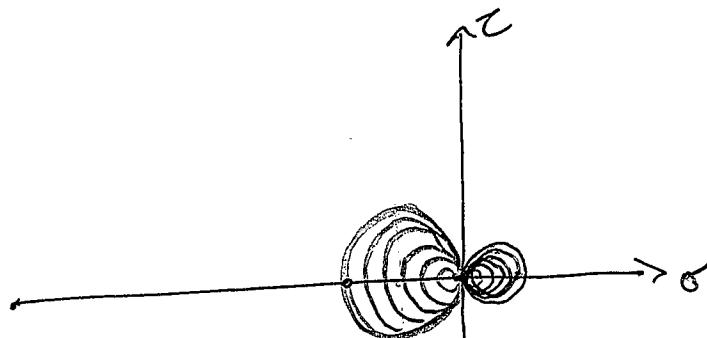
(4) A $\rightarrow (2.25, 2)$ m. $E_{ij}(2.25, 2) \rightarrow \dots$ $\xrightarrow{\text{flag}} \delta_{ij} \dots$

Kritiko Aroxios Mahr-Coulomb (Brittle)



$$\sigma_{cr} = \frac{F_{comp}}{A}$$

$$\sigma_{ij} = \begin{pmatrix} -\frac{F_{comp}}{A} & 0 \\ 0 & 0 \end{pmatrix}$$

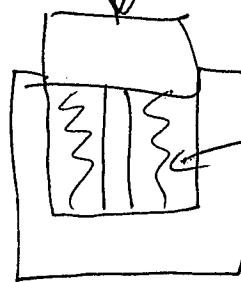


→ Eiflos → so follows the Tension (fracture condition)

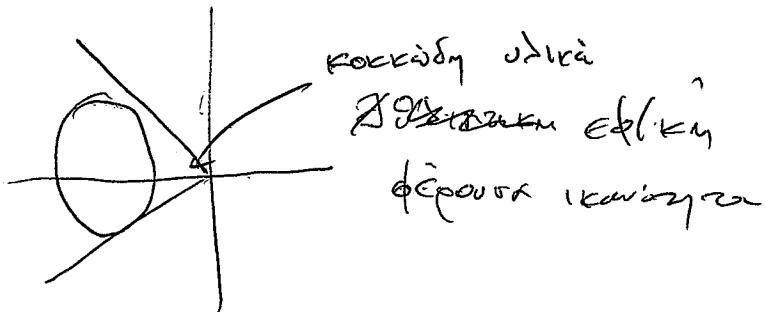
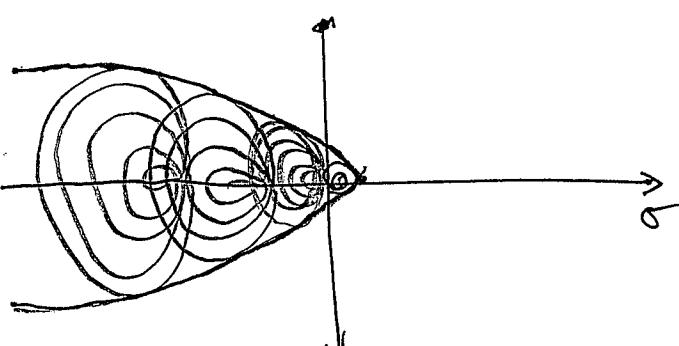
$$\sigma_{ij} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & P \end{pmatrix}$$

Der Bruchkriterium ist $\sigma_1 = \sigma_{cr}$
Sobald es $\sigma_1 > \sigma_{cr}$ ist, so
nimmt $F_{comp} = F_{comp}(A, \sigma_1)$

→ hydrostatic → $\sigma_{ij} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & P \end{pmatrix}$



$\sigma_{ij} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & P \end{pmatrix}$
so $\sigma_1 = P$,
 $\sigma_{ij} = \begin{pmatrix} P & 0 & 0 \\ 0 & -P & 0 \\ 0 & 0 & P \end{pmatrix}$
so $\sigma_1 = P$
so $P = \sigma_{cr}$
so $\sigma_{ij} = \begin{pmatrix} \sigma_{cr} & 0 & 0 \\ 0 & -\sigma_{cr} & 0 \\ 0 & 0 & \sigma_{cr} \end{pmatrix}$



→ Δύο κυλινδρικά δοκίμια συρροής.

$$D = 15 \text{ cm} \quad \textcircled{1} \rightarrow \text{Compression} \quad P_{sp}^C = 45 \text{ t.m}$$
$$L = 30 \text{ cm.} \quad \textcircled{2} \rightarrow \text{Brazilian}\newline \text{Tension} \quad P_{sp}^{Br} = 15 \text{ t.m.}$$

→ Το ωδικό περιγράφεται με δεσμητική τριγωνική σχήματα.

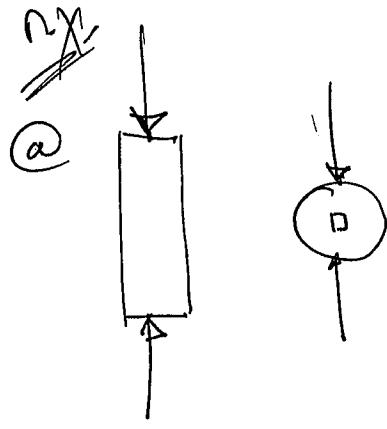
ⓐ Αυτοχώριο σε σερέψη?

ⓑ Η αρδική τάση που δεν προσδέσει αυτοχώριο αν το δοκίμιο
υφίσσεται παραπλέυρη ηλεκτρού 10 atm. $\rightarrow 1 \text{ atm} = 100 \text{ kPa}$
 $\hookrightarrow 10 \text{ atm} = 1 \text{ MPa.}$

ⓒ Η παραπλέυρη ηλεκτρού που ανατίθεται είναι η
τύπου λοτοχύσης, όπου αλιεύεται τάση λοτοχύσης με P_{sp}^{Comp} .

ⓓ Η διεργάδει τατα μέσο $\sigma_{ij} = \begin{pmatrix} 35 & 20 \\ 20 & -70 \end{pmatrix} \text{ MPa}$ είναι
επιτρεπτός

ⓔ Οι στοιχίων $\sigma_{ij} = \begin{pmatrix} 40 & 25 & 0 \\ 25 & 30 & 0 \\ 0 & 0 & 60 \end{pmatrix} \text{ MPa.}$



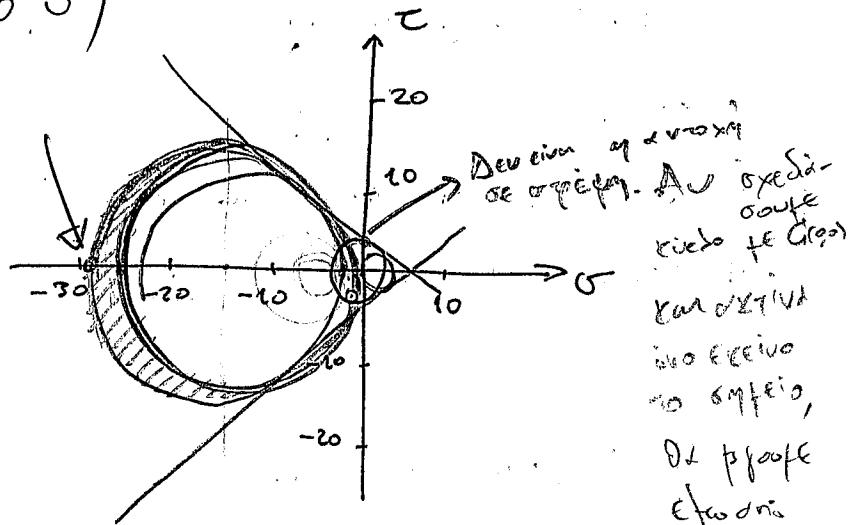
$$\sigma_{ij}^{\text{comp}} = \begin{pmatrix} -25.5 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\sigma_{ij}^{\text{tors}} = \begin{pmatrix} 2.1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$P_0 = 45 \text{ fm} \quad P_{Br} = 15 \text{ fm}$$

$$\sigma_{B3} = \frac{P_{B1}}{A_0} = \frac{45 \times 10^4 \times 4}{\pi \cdot 0.15^2} = 25.5 \text{ MPa}$$

$$P_{ef}^{\text{Br}} = \frac{2P_{Br}}{\pi D L} = \frac{2 \cdot 15 \times 10^4}{\pi \cdot 0.15 \times 0.30} \approx 2.1 \text{ MPa}$$



$\Rightarrow G(0,0)$ kan va
Expansion træs ka-
kia sammensæt-
ningsfaktoren

$$\textcircled{P} \quad \sigma_{ij} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & -P \\ 0 & 0 & -P \end{pmatrix} \quad \text{Yield 3-D}\\ \text{represents}$$

$$\Rightarrow \sigma_{ij} = \begin{pmatrix} \sigma_1 & 0 \\ 0 & 1 \end{pmatrix}$$

For centrifugal stress: $\sigma_1 = -27.5 \text{ MPa}$ ($\Rightarrow f$) kules for vægt $G = \frac{\rho g}{2}$.
Trial + error opnem o kules ved at finne det som representerer.

$$\textcircled{Q} \rightarrow \sigma_{ij} = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & -P_2 & 0 \\ 0 & 0 & -P_2 \end{pmatrix} \rightarrow \text{Autosyntaf for } f.$$

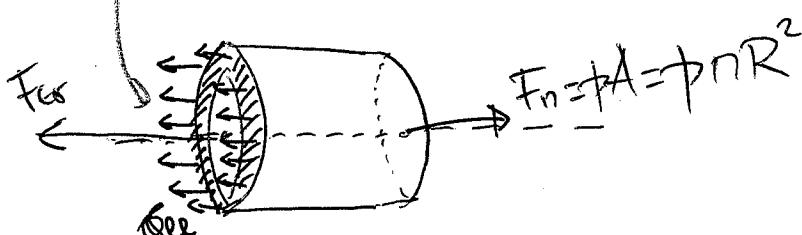
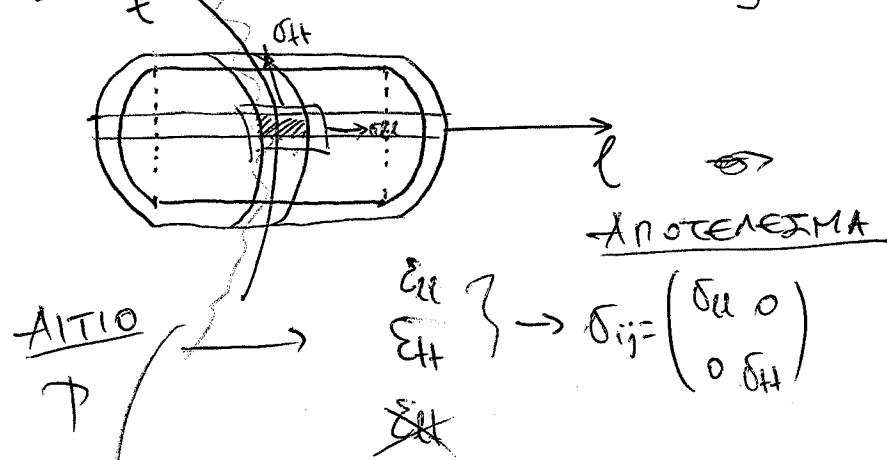
(40)

⑤ $\sigma_{ij} = \begin{pmatrix} 35 & 20 \\ 20 & -70 \end{pmatrix} \rightarrow$ Σχεδιασμός του κύλινδρου. Αν τις φέρει τις περιβολούσες, την επιτρέπεται.

⑥ $\sigma_{ij} = \begin{pmatrix} 40 & 25 \\ 25 & 30 \\ 0 & 0 \end{pmatrix} \rightarrow$ Φεγγίδα 3-D.
 $\sigma_{ij} = \begin{pmatrix} 60.5 & & \\ & 39.5 & \\ & & 60 \end{pmatrix}$

Πάρτε ως για τη σχεδιαστική διαφορά \Rightarrow Σχεδιασμός του κύλινδρου +
 Εξισώσεις της συνοριακής σύνθετης στήλης + περιβολούσες

[ΧΕΙΡΟΤΟΠΟΙΚΑ ΔΟΧΕΙΑ ΠΗΕΤΗΣ]
 ΛΕΒΗΤΕΣ



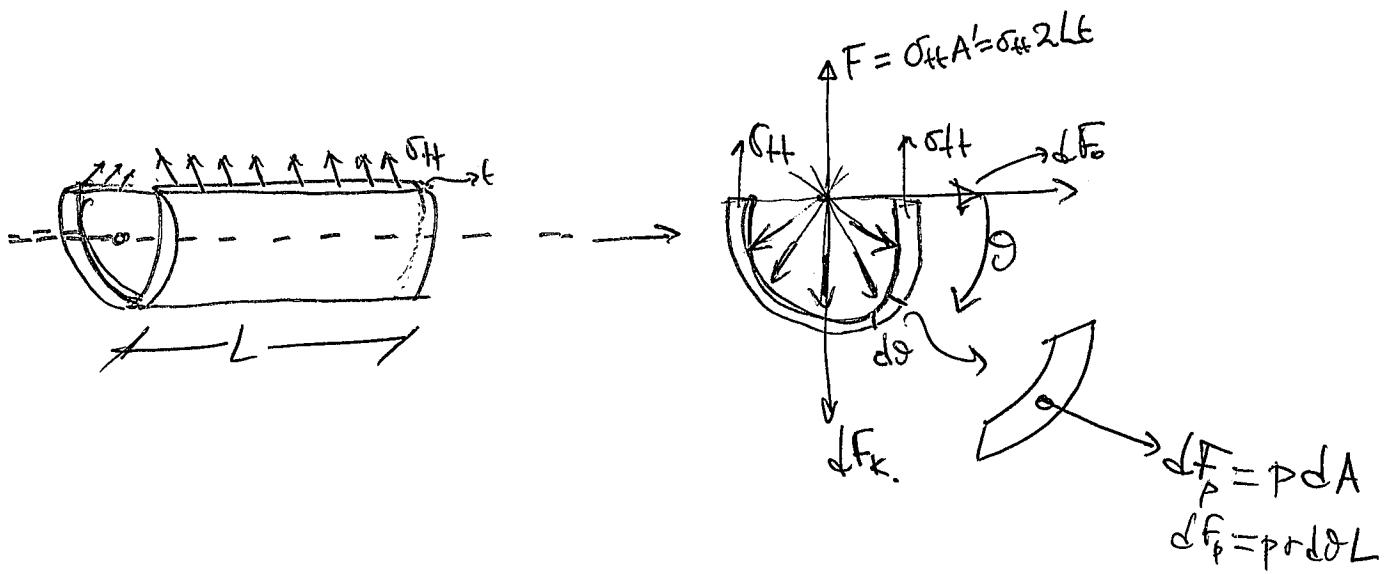
$\hookrightarrow F_{ext} = \sigma_{ll} \cdot A = \sigma_{ll} \cdot (2\pi R t) = \sigma_{ll} 2\pi R t.$

$nR_o^2 - nR_i^2 = n(R_o - R_i)(R_o + R_i)$

$F_{ext} = F_n.$

$\hookrightarrow p\pi R^2 = \sigma_{ll} 2\pi R t \Rightarrow$

$\boxed{\sigma_{ll} = \frac{pR}{2t}}$



$$dF_k = dF \sin \theta \rightarrow [dF_k = \rho R L \sin \theta d\theta]$$

$$dF_p = \rho r d\theta$$

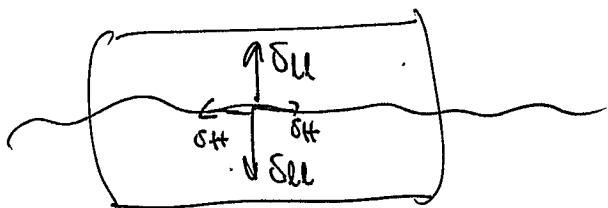
$$dF_p = \rho R L d\theta$$

$$\int_0^\pi dF_k = F \Rightarrow \int_0^\pi \rho R L \sin \theta d\theta = \sigma_{tt} 2 \pi R L t \Rightarrow \rho R \cos \theta \Big|_0^\pi = 2 \sigma_{tt} t \Rightarrow \cancel{\rho R t} = 2 \sigma_{tt} t$$

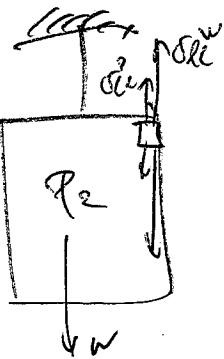
$\sigma_{tt} = \frac{\rho R}{E} = 2000$

$$\left[\delta_{ij} = \begin{pmatrix} \frac{\rho R}{2t} & 0 \\ 0 & \frac{\rho R}{t} \end{pmatrix} \right]$$

Kuivudustas kiepsytos oni otsikko oikea tapahtum tampa -
deltaxit teki pieni. Na ongelmaa ei voinut.

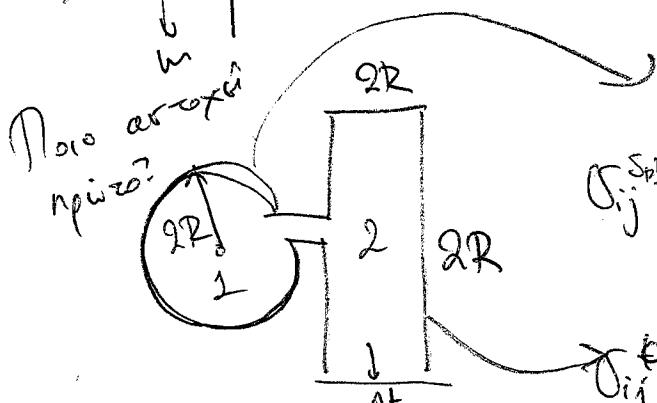


n.X. Ασύρματη πίεση;



$$\sigma_{xx} = \frac{P_1}{t}$$

$P_2 < P_1$
το γ δρα ανακοπής



$$\sigma_{yy} = \frac{P_0}{t} = \left(\frac{P_0}{t} \right) = \left(\frac{P_0 R}{t} \right)$$

$$\sigma_{yy} = \left(\frac{P_0}{t} \right) = \left(\frac{P_0 R}{2t} \right) = \left(\frac{P_0 R}{2t} \right)$$

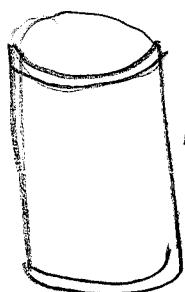
Mises $\frac{\sigma_{yy}}{\sigma_{yy}}$ $\left(\frac{P_0 R}{t} \right)^2 \leq \sigma_y^2$

Mises $\frac{\sigma_{yy}}{\sigma_{yy}}$ $\left(\frac{P_0 R}{2t} \right)^2 + \left(\frac{P_0 R}{t} \right)^2 - \left(\frac{P_0 R}{2t} \right) \left(\frac{P_0 R}{t} \right) \leq \sigma_y^2$

$$(\sigma_{xx} - \sigma_{yy})^2 + 3\sigma_{xy}^2$$

$$\frac{3}{4} \left(\frac{P_0 R}{t} \right)^2 \leq \sigma_y^2$$

n.X.



$$\begin{aligned} R &= 25 \text{ cm} \\ t &= 1 \text{ mm} \\ L &= 2 \text{ m} \\ \rho &= 80 \text{ kN/m}^3 \end{aligned}$$

$$E = 200 \text{ GPa}$$

$$\nu = 0.3$$

$$\sigma_y = 100 \text{ MPa}$$

πάρουσιας +
 $\sigma_p = 5 \text{ MPa}$
μέτα +

1) Αγνωστες παρατητικοι φυσικοι τα τα βαθυ τω κανονιων.

① Κρίση συγγένειας

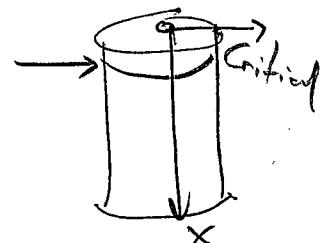
② Η προσδιοριση της στρεσσος σ_{ij} .

③ Ολοκληρωτικη μηδενιαση.

④ Η επιδρση τω τελικοι τιμοι τω διαφεροντων λεπτων.

$$\textcircled{a} M_f \rightarrow \begin{pmatrix} 0 & \frac{M_f R}{I_p} \\ \frac{M_f R}{I_p} & 0 \end{pmatrix} \quad \dot{\phi} \rightarrow \begin{pmatrix} \frac{P R}{2f} & 0 \\ 0 & \frac{P R}{2f} \end{pmatrix} \quad f \propto y, z.$$

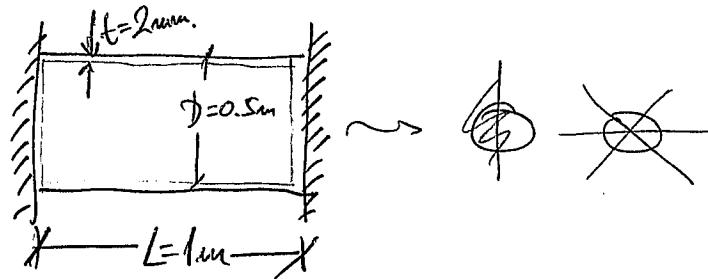
$$f \rightarrow \begin{pmatrix} -w(x) & 0 \\ \frac{-w''(x)}{2nRt} & 0 \end{pmatrix} \quad \times \text{ tecpical case} \quad \cancel{\text{no w}} \quad \rightarrow$$



\textcircled{b} Αφού καθορίσατε τη διάταξη συνειδ. ($x=0$) νέων διαδικασιών
 \Rightarrow δικυριούντε το βαθός!

\textcircled{c} με Ρεάλα

$$\boxed{B} \quad E = 120 \text{ GPa.} \\ \nu = 0.32 \\ \sigma_g = 80 \text{ MPa}$$



$$\text{ring } A^{\text{ring}} = A^{\text{out}} - A^{\text{in}} = \pi R_{\text{out}}^2 - \pi R_{\text{in}}^2 = \pi (R^{\text{o}}_0 R^{\text{i}}_0)^+ (R^{\text{o}} + R^{\text{i}}) = 2\pi R t.$$

$$\text{γ.γ. flode } \varepsilon_{ll} = \frac{1}{E} (\sigma_{ll} - \nu \sigma_{tt}) \rightarrow \varepsilon_{ll} = 0 \quad \sigma_{ll} = \nu \sigma_{tt} \quad \cancel{\text{Επίπεδη}} \quad F = \nu R^2 n (1-2\nu) \quad F = 0.707 \nu [N].$$

! Διάφορες Τιμολογίες! $\varepsilon_{ll} = 0 \rightarrow \sigma_{ll} = 0$!

\textcircled{d} Η εργασία για την πρώτη φάση σε σημείο του στρώματος
~~είναι~~ $S.F. = 1.5$. \rightarrow Η μεγαλύτερη καταστρεψη στο στρώμα.

$$\sigma_{ij} = \begin{pmatrix} \frac{P R}{2f} & 0 \\ 0 & \frac{P R}{2f} \end{pmatrix} \quad \text{Ηλεκτρική στρέση.}$$

$$\sigma_{ll} = \nu \sigma_{tt} \quad \sigma_{ij} = \begin{pmatrix} \nu \sigma_{tt} & 0 \\ 0 & \sigma_{tt} \end{pmatrix}$$

$$\text{Hiles: } \sigma_I^2 + \sigma_{II}^2 - \sigma_I \sigma_{II} \leq \left(\frac{\sigma_y}{1.5} \right)^2$$

$$\frac{\sigma_{xx}^2 + \sigma_{yy}^2 - V\sigma_{xx}\sigma_{yy}}{2} \leq \frac{P}{E} \rightarrow P_{max} \leq 0.48 \text{ MPa}$$

① $P = 0.54 \text{ MPa}$

$$\sigma_{ij} = \begin{pmatrix} \frac{V+R}{E} & 0 \\ 0 & \frac{P-R}{E} \end{pmatrix} \xrightarrow{P=0.5 \text{ MPa}} \sigma_{ij} = \begin{pmatrix} 40 & 0 \\ 0 & 125 \end{pmatrix} \text{ MPa} \rightarrow \epsilon_{ij} = (?)$$

$$\frac{0.5 \times 10^6 \times 0.5}{2 \times 10^3} \rightarrow \sigma = \frac{\frac{1}{4} \times 10^6}{\frac{2 \times 10^3}{4}} = 125 \text{ MPa}$$

$$\epsilon_{xx} = \frac{1}{E} (\sigma_{xx} - V\sigma_{yy}) \xrightarrow{V=0} \epsilon_{xx} = \frac{1}{E} (\sigma_{xx} - 0) = \frac{1}{120} \times 10^9 (125 - 0)$$

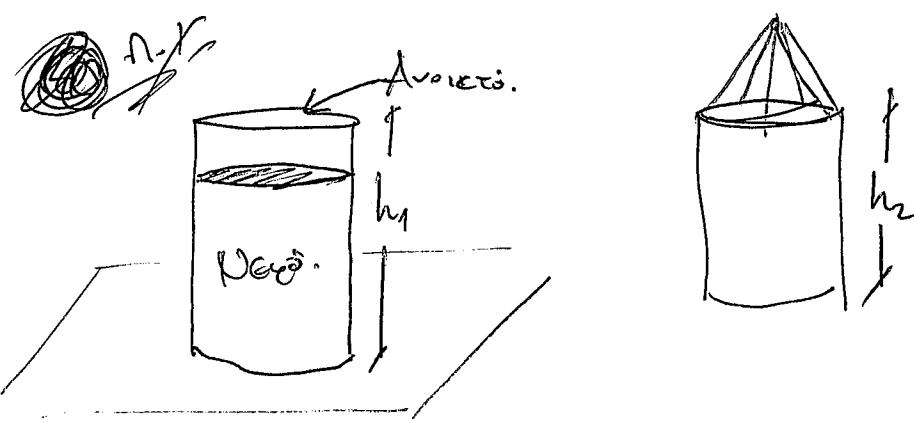
$$= \frac{1}{120} \times 10^9 (125 \times 10^6 - (0.32)^2 \times 125 \times 10^6) = 0.708 \times 10^{-3} = 0.708 \times 10^{-3}$$

$$\epsilon_{xx} = \frac{P - P_0}{E} = \frac{0.5 - P_0}{120} = 0.422 \times 10^{-3}$$

② $P = P_{max}$

$$\epsilon_{ij} = \begin{pmatrix} 0 & 0 \\ 0 & 0.422 \end{pmatrix} \times 10^{-3} \rightarrow \sigma_{ij} = 0.422 \times 10^{-3}$$





Diferenças →

σ_y
Mises

$$\int \rightarrow h_1 = 2m \text{ (figura) } \text{ e } \text{ (equação).}$$

Draw → calculate $h_2 = ?$

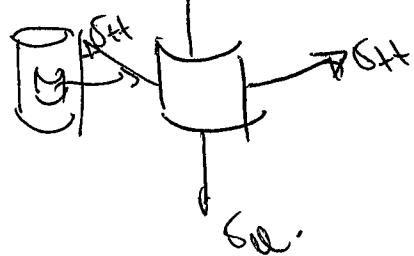
$$\begin{aligned} \sigma_{ij} &= \begin{pmatrix} \sigma_{xx} & 0 \\ 0 & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_{yy} \end{pmatrix} \\ \sigma_{ij} &= \begin{pmatrix} \sigma_{xx} \\ -\frac{\partial W(x)}{\partial R} \end{pmatrix} \quad \sigma_{ij} = \begin{pmatrix} \frac{\partial W(x)}{2\pi R t} & 0 \\ 0 & \sigma_{yy} \end{pmatrix}. \end{aligned}$$

$$\text{S.t. } \text{for given scaling} \quad \sigma_{yy} = \frac{p(2)R}{t} \rightarrow p = \rho z.$$

$$\text{Kilometer} \rightarrow \sigma_{yy} = \frac{ph_2 R}{t}.$$

$$\sigma_{yy} = \sigma_y \cdot \boxed{h_2 = \frac{\sigma_y t}{pR}}.$$

Les AEP. (a) L'elasto-sifon → tocas káces.



$$\sim \sigma_{ij} = \begin{pmatrix} \sigma_{xx} & 0 \\ 0 & \frac{ph_2 R}{t} \end{pmatrix}$$

$$\sigma_{ee} = \frac{W}{A} = \frac{\pi R^2 h_2 p}{2\pi R t}$$

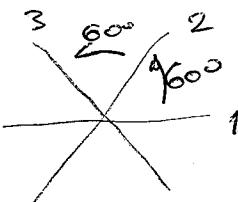
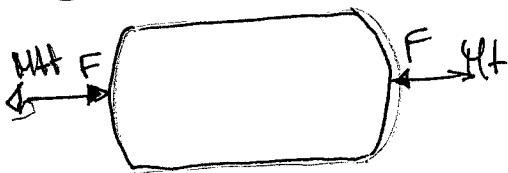
$$\hookrightarrow \sigma_{ij} = \begin{pmatrix} \frac{\pi R h_2}{2t} & 0 \\ 0 & \frac{ph_2 R}{t} \end{pmatrix}$$

(46)

$$\sigma_{\text{out}}^2 + \sigma_{\text{eff}}^2 - \cancel{\sigma_{\text{out,eff}}} \leq \sigma_y^2 \rightarrow \frac{3}{4} \frac{e^2 R^2}{f^2} h_2^2 \leq \sigma_y^2$$

$$\boxed{h_2 = \frac{2 \sigma_y f}{\sqrt{3} p R}} \rightarrow \frac{h_1}{h_2} \rightarrow \boxed{h_2 = \frac{4}{\sqrt{3}} \text{ m}}$$

(1) (2)



$$\begin{aligned}\varepsilon_1 &= 2 \times 10^{-4} \\ \varepsilon_2 &= 4 \times 10^{-4} \\ \varepsilon_3 &= 12 \times 10^{-4}\end{aligned}$$

(3) $\varepsilon_{xx} = \varepsilon_2 \quad \varepsilon_{yy} = 2 \times 10^{-4}$

$$\varepsilon_{xy} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \cos(120^\circ) + \varepsilon_{xy} \sin(120^\circ)$$

~~(3)~~ $\varepsilon_{xy} = \frac{4 \times 10^{-4}}{2} = 2 \times 10^{-4}$ ignore $\varepsilon_{xx}, \varepsilon_{yy}$ { $\varepsilon_{yy} = \dots$ } $\varepsilon_{xy} = \dots$

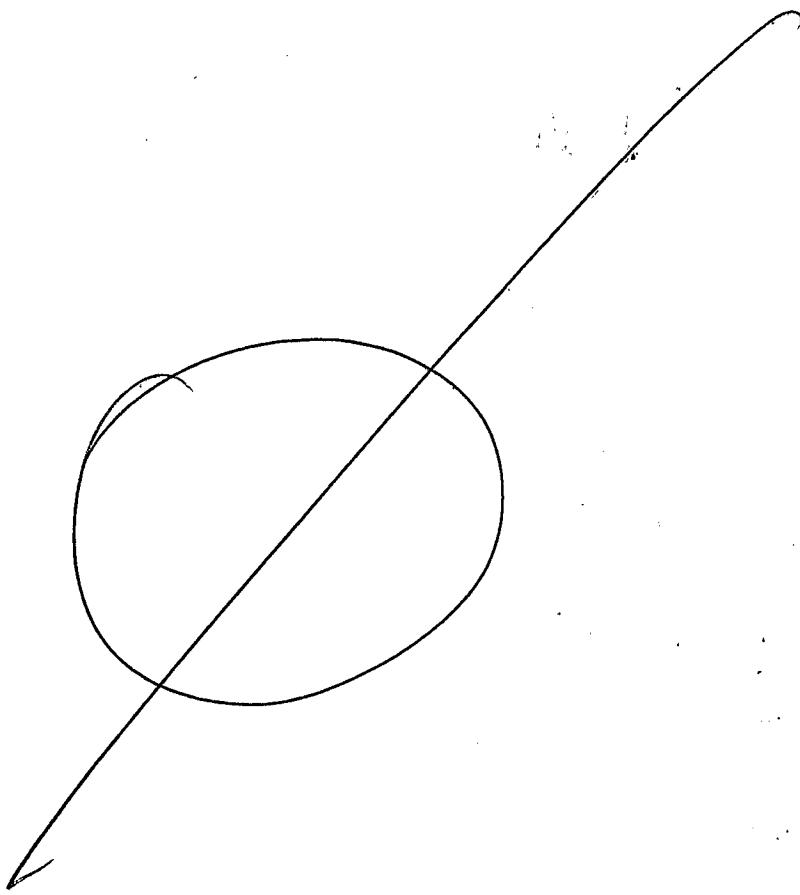
$$\varepsilon_{ij} = \begin{pmatrix} 2 & -4.62 \\ -4.62 & 10 \end{pmatrix} \times 10^{-4} \xrightarrow{\text{rev. v. Hooke.}}$$

(4) $\sigma_{ij} = \begin{pmatrix} 110 & -21 \\ -21 & 233 \end{pmatrix} \text{ MPa} \rightarrow \theta_p = 24.5^\circ \rightarrow \sigma_{I,II} = \begin{pmatrix} 265.4 & 0 \\ 0 & 77.6 \end{pmatrix} \text{ MPa.}$

(5) $T = \frac{M_f R}{I_p} \rightarrow 71 \times 10^6 = \frac{M_f \cdot 124}{\frac{124}{2}} \rightarrow M_f = 55.76 \text{ kNm.}$

$$\sigma_{\text{eff}} \leftrightarrow \phi \rightarrow \sigma_{\text{eff}} = \frac{P R}{I} = 233 \times 10^6 \rightarrow \phi = \dots$$

$$\sigma_{\text{eff}} \leftrightarrow P + F \rightarrow \text{fixed w.r.t. } P \Rightarrow F \text{ picked.}$$



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